MATH 12
LOGARITHMS
PROBLEM SET

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FINAL SCORE: 28/28

SHOW ALL YOUR WORK AND USE PROPER MATHEMATICAL SYMBOLS
UNLESS OTHERWISE STATED, FINAL ANSWERS MUST BE EXACT (NO DECIMALS) AND SIMPLIFIED TO LOWEST TERMS

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1. Decide whether each of the following statements are TRUE or FALSE and write your answer in the space provided to the right. (4 marks)

a) $16^{a+1} \div 8^{3a-1} = 2^{7-5a}$

$2^{4a+4} \div 2^{9a-3} = 2^{7-5a}$

$4a+4-9a+3 = 7-5a$

$-5a+7 = -5a+7$

True.

b) If $f(x) = \log_3(x-4)$, then $f^{-1}(x) = 3^x + 4$

$y = \log_3(x-4) \rightarrow x = \log_3(y-4) \rightarrow 3^x = y-4$

$y = 3^x + 4$

True.

c) Every logarithmic function has a range of all real numbers

True.

d) When rounded to the nearest hundredth, $\log_2 7 = 0.36$

$\frac{\log 7}{\log 2} \cong 2.81$

False.

2. Use your knowledge of exponents and logarithms to answer the following questions. (4 marks)

a) Write $\log_7 350 - 1 + \frac{3}{2}\log_7 9$ as a simplified single logarithm.

$\log_7 350 - \log_7 7 + \frac{3}{2}\log_7 9$

$\log_7 350 - \log_7 7 + \log_7 (\sqrt{9})^3$

$\log_7 350 - \log_7 7 + \log_7 27 \rightarrow \log_7 \left(\frac{350 \cdot 27}{7}\right) = \boxed{\log_7 1350}$

b) Simplify: $(a^{-2\log_a b})(\log_a a^b)$

$(a^{\log_a b^{-2}})(b)$

$(b^{-2})(b) = b^{-2} \cdot b = \boxed{\frac{1}{b}}$

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c) Write $\log_x y = z$ in exponential form.

$x^z = y$

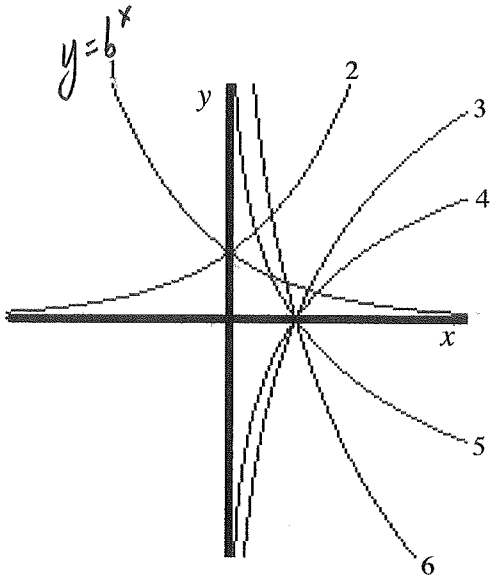
d) If $f(x) = \log_{\sqrt{7}} x$, then what is the equation of $y = f^{-1}(x)$?

$y = \log_{\sqrt{7}} x \rightarrow x = \log_{\sqrt{7}} y$

$y = (\sqrt{7})^x$ or $y = 7^{\frac{x}{2}}$

3. **On the grid to the right, $y = b^x$ is represented by graph 1.** (2 marks)

∴ b must be a fraction.



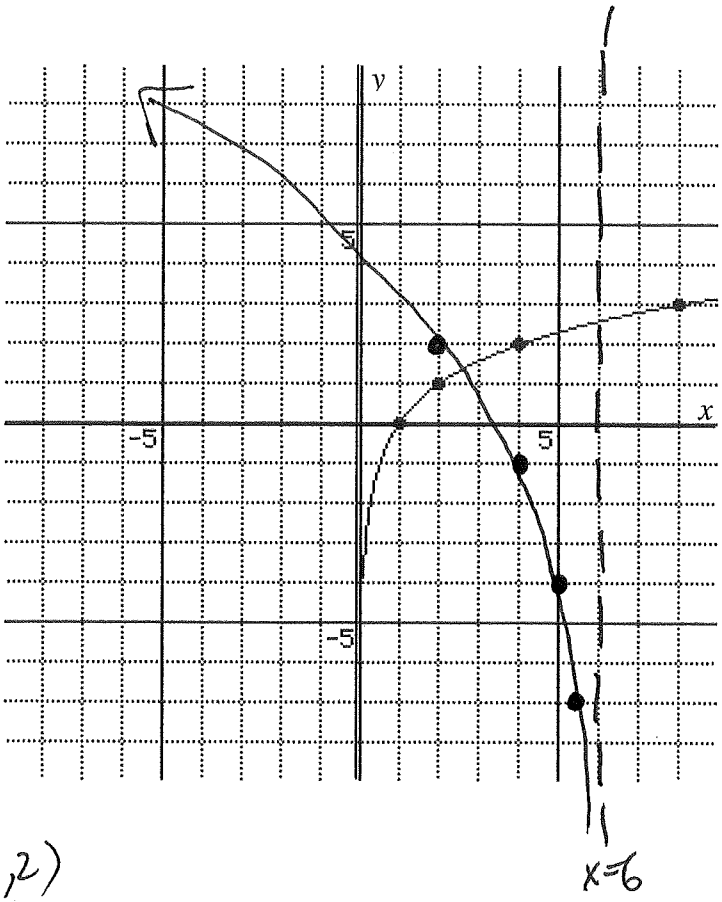
a) Which is the graph of $y = \left(\frac{1}{b}\right)^x$?

Graph #2

b) Which is the graph of $y = \log_b x$?

Graph #5

4. **The grid to the right shows the graph of $y = \log_2 x$. On the same grid, sketch the graph of $y + 7 = 3\log_2(-2x + 12)$.** (3 marks)



$y = 3\log_2 -2(x-6) - 7$

asymptote now at $x = 6$

$(x, y) \rightarrow \left(\frac{-x}{2} + 6, 3y - 7\right)$

$(1, 0) \rightarrow (5.5, -7)$

$(2, 1) \rightarrow (5, -4)$

$(4, 2) \rightarrow (4, -1)$ $(8, 3) \rightarrow (2, 2)$

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5. **Use your graphing calculator to find the solution(s) to the equation $\log_3 x + 3 = 3^x$. If necessary, round your answer(s) to the nearest hundredth.** (2 marks)

graph $\log_3 x = 3^x - 3$ & look for intersect pt.

$\frac{\log x}{\log 3} = 3^x - 3$

y_1 y_2 & find intersect on calculator.

intersect pt. at $x = 1$ & 0.13 (zoom on calc.)

6. **What is the domain of the function $y = 2\log_x(x+5)$?** (2 marks)

D: $x > 0$ except $x \neq 1$.

$y = \log_x(x+5)^2$ or $y = 2\left(\frac{\log(x+5)}{\log x}\right)$

$x \neq 1, x > 0$

7. **What is the *y*-intercept of the function $y - 16 = -3\left(\frac{1}{4}\right)^{5x}$?** (1 mark)

let $x = 0$

$y - 16 = -3\left(\frac{1}{4}\right)^0$

$y - 16 = -3(1)$

$y = 16 - 3$

y-int at $y = 13$

8. **Algebraically find the EXACT solution(s) for each of the following equations.** (6 marks)

a) $\left(\frac{1}{8}\right)^{x-3} = 2(16^{x+1})$

$2^{-3x+9} = 2^1 \cdot 2^{4x+4}$

$-3x + 9 = 4x + 5$

$4 = 7x$

$\frac{4}{7} = x$

b) $4^x = 3^{x-1}$ log

$x\log 4 = (x-1)\log 3$

$x\log 4 = x\log 3 - \log 3$

$x\log 4 - x\log 3 = -\log 3$

$x(\log 4 - \log 3) = -\log 3$

$x = \frac{-\log 3}{\log 4 - \log 3}$

$= \frac{\log 3}{\log 3 - \log 4}$

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c) $\log_7(x+1) - 1 = \log_7(x-5)$

$\log_7(x+1) - \log_7 7 = \log_7(x-5)$

$\log_7\left(\frac{x+1}{7}\right) = \log_7(x-5)$

$7\left[\frac{x+1}{7} = x-5\right]$

$x+1 = 7x-35$

$36 = 6x$

$\boxed{6 = x}$

. . **Rounded to the nearest tenth, algebraically find the solution(s) of $\log x + \log_2 x = 1$.** (2 marks)

$\log x + \frac{\log x}{\log 2} = 1$

common denominator $\log 2$

$\left[\frac{\log 2 \log x}{\log 2} + \frac{\log x}{\log 2} = 1\right]$

$\log 2 \log x + \log x = \log 2$

Omitted

$\log x(\log 2 + 1) = \log 2$

$\frac{\log x(\log 2+1)}{(\log 2+1)} = \frac{\log 2}{\log 2+1}$

$\log x = \frac{\log 2}{\log 2+1}$

$10^{\frac{\log 2}{\log 2+1}} = x \rightarrow \boxed{x \cong 1.7}$

or $\log x + \frac{\log x}{\log 2} = 1$

$\frac{\log x\left(1+\frac{1}{\log 2}\right)}{1+\frac{1}{\log 2}} = \frac{1}{1+\frac{1}{\log 2}}$

$x = 10^{\frac{1}{1+\frac{1}{\log 2}}}$

9. **Due to their current losing streak, fan support of the Vancouver Canucks has been decreasing by 1.77% every week. In fact, a recent poll by the Vancouver Sun showed that only 67% of the people of Vancouver still consider themselves fans. At this rate, how many days will it take until the fan support in Vancouver is LESS THAN 50% ? (Round your final answer to the nearest day.)** (2 marks)

$y = Ab^x$

$\frac{50}{67} = \frac{67\left(1 - \frac{1.77}{100}\right)^{\frac{x}{7}}}{67}$

$\frac{50}{67} = (1 - .0177)^{\frac{x}{7}}$

$\frac{50}{67} = (.9823)^{\frac{x}{7}}$

$\log_{.9823}\left(\frac{50}{67}\right) = \frac{x}{7}$

$7\log_{.9823}\left(\frac{50}{67}\right) = x$

$7 \cdot \frac{\log\left(\frac{50}{67}\right)}{\log .9823} = x$

$114.7 \cong x$

days

$\boxed{115 \text{ days}}$

* to get to under 50% support.

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