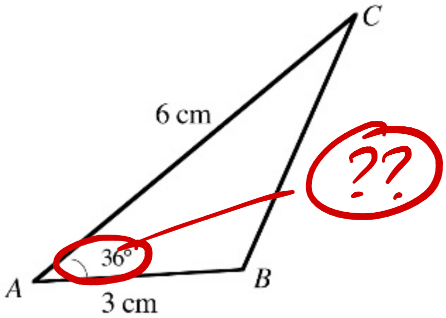
Trigonometry - Sine and Cosine Laws Lesson #3:
The Cosine Law

Introduction

Consider triangle ABC in which $\angle A = 36°$, $AB = 3$ cm and $AC = 6$ cm. What happens when you try to apply the sine law to determine the length of BC?

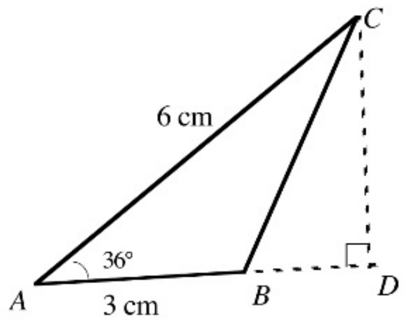


In the example above, where we are given the length of two sides and the contained angle, the sine law is **not** applicable.

no angle/opp. side relationship.

We can find the length of BC by making a right triangle BCD in the diagram below and using SOHCAHTOA to determine the lengths of CD and AD.

Determine the lengths of CD and AD to the nearest hundredth of a cm, and show how these lengths can be used to determine the length of BC to the nearest tenth of a cm.



The method above is time consuming.
The length of BC can be determined in one step by using the **cosine law**.

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The Cosine Law

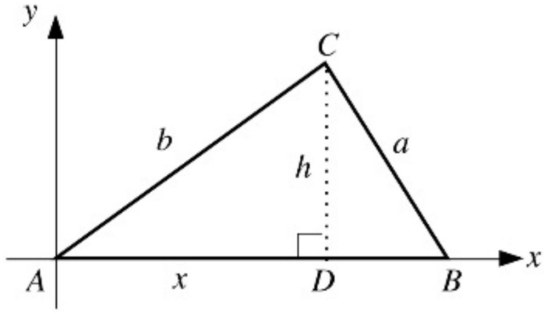
In every triangle ABC, $a^2 = b^2 + c^2 - 2bc\cos A$.

$b^2 = a^2 + c^2 - 2ac\cos B$

* $c^2 = a^2 + b^2 - 2ab\cos C$

Proof of the Cosine Law

- The diagram shows triangle ABC placed with base AB on the x–axis and A at the origin.
- The line CD is drawn perpendicular to AB and is h units in length.
- $AD = x$ units so $DB = c - x$ units.



Complete the following work to show that $a^2 = b^2 + c^2 - 2bc\cos A$.

In ΔADC, $\cos A = \dfrac{AD}{AC} = \dfrac{x}{b}$

so $x =$

In ΔBDC, $BC^2 = CD^2 + DB^2$

$a^2 = h^2 + (c - x)^2$

$a^2 = h^2 + c^2 - 2cx + x^2$

$a^2 = (h^2 + x^2) + c^2 - 2cx$

$a^2 = \quad + c^2 - 2c(\quad\quad)$

$a^2 = b^2 + c^2 - 2bc\cos A.$

By placing AC and then BC on the x–axis, similar equations can be derived.

$b^2 = c^2 + a^2 - 2ca\cos B$ $\qquad$ $c^2 = a^2 + b^2 - 2ab\cos C$

This version of the cosine law can be used in any triangle if we are given the lengths of two sides and the contained angle (SAS).

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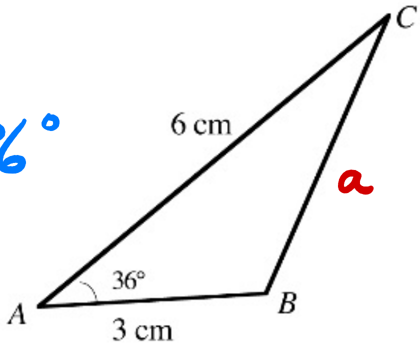
Consider the ΔABC from Class Ex. #1 in which $\angle A = 36°$, $AB = 3$ cm, and $AC = 6$ cm. Determine the length of BC, to the nearest tenth of a cm, using the cosine law.

$$a^2 = 6^2 + 3^2 - 2(6)(3)\cos 36°$$

$$a = \sqrt{45 - 36\cos 36°}$$

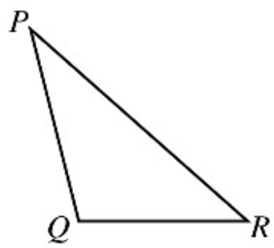
$$a = 4.0\text{cm}$$



Consider triangle PQR shown.

a) Complete the cosine law for calculating side q. $q^2 =$

b) Calculate, to the nearest tenth of a cm, the length of the third side of ΔPQR if $QP = 1.7$ cm, $QR = 3.1$ cm, and $\angle PQR = 110°$.



Bellevue is 30 km north of Ayr and Churchville is 18 km northwest of Ayr. Calculate the distance between Bellevue and Churchville to the nearest km.

Complete Assignment Questions #1 - #4

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Alternative Form of the Cosine Law

The equation $a^2 = b^2 + c^2 - 2bc \cos A$

can be rearranged to the form $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

This form of the cosine law can be used to determine any angle in a triangle when we are given the length of all three sides (SSS).

Class Ex. #5

Complete the following for triangle *ABC*.

a) $\cos B =$ **b)** $\cos C =$

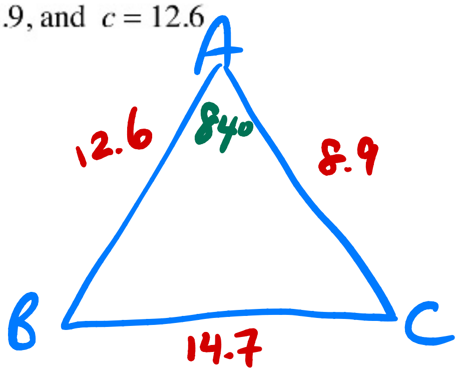
Class Ex. #6

Determine the largest angle in ΔABC if $a = 14.7$, $b = 8.9$, and $c = 12.6$

$\angle A$

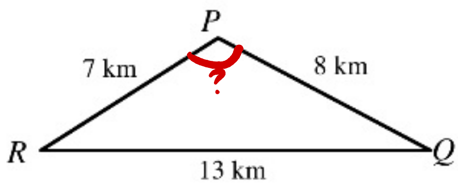
$$\cos A = \frac{8.9^2 + 12.6^2 - 14.7^2}{(2(8.9)(12.6))}$$

$$\cos^{-1}(\text{ANS}) = \angle A = 84^\circ$$



Class Ex. #7

Two ships set sail from port, *P*, heading in different directions. The first ship sails 7 km to *R* and the second ship sails 8 km to *Q*. If the distance between *R* and *Q* is 13 km, determine the angle between the directions of the two ships.



$$\cos P = \frac{7^2 + 8^2 - 13^2}{(2(7)(8))}$$

$$\cos^{-1}(\text{ANS}) = P = \boxed{120^\circ}$$

$\boxed{\#2\text{-}5, 7, 9, 10}$

Complete Assignment Questions #5 - #11 and the Group Investigation.

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