Rational Expressions and Equations Lesson #1:
Simplifying Rational Expressions - Part One

Rational Expressions

In previous courses we have learned that the quotient of two integers $\left(\text{eg. } \dfrac{4}{7}\right)$ is a **rational number**.

Similarly, the quotient of two polynomials $\left(\text{eg. } \dfrac{x+7}{x+8}\right)$ is called a **rational expression**.

A single variable **rational expression** is an algebraic fraction in one variable in which the numerator and denominator are both polynomials $\left(\text{eg. } \dfrac{x-3}{x^2+1}, \dfrac{7}{2y+5}\right)$.

In this unit, we will learn how to add, subtract, multiply and divide rational expressions. We will also learn how to solve problems involving rational equations.

The key to success in operating with rational expressions in this unit lies in our ability to factor polynomials.

Recall the following methods for factoring polynomial expressions:

i) Greatest Common Factor

ii) Difference of Squares

iii) Factoring Trinomials by Inspection

iv) Factoring Trinomials by Decomposition

v) Grouping

Investigating Equivalent Forms of a Rational Expression

Consider the rational expressions

$\frac{2x+2}{x^2+3x+2}$ and $\frac{2}{x+2}$.

a) Complete the table.

Value of x	Value of $\frac{2x+2}{x^2+3x+2}$	Value of $\frac{2}{x+2}$
0	1	1
1	$\frac{2}{3}$	$\frac{2}{3}$
2	$\frac{1}{2}$	$\frac{1}{2}$
3	$\frac{2}{5}$	$\frac{2}{5}$
4	$\frac{1}{3}$	$\frac{1}{3}$

b) What can we say about the values of the rational expressions $\frac{2x+2}{x^2+3x+2}$ and $\frac{2}{x+2}$, when x is replaced by 0, 1, 2, 3, or 4? equal

c) The expressions $\frac{2x+2}{x^2+3x+2}$ and $\frac{2}{x+2}$ are known as **equivalent forms** of a rational expression. To explain why they are equivalent, read the following procedure and then complete the factoring and simplification after step ii).

i) Write the numerator, $2x+2$, and the denominator, x^2+3x+2 in factored form.

ii) Reduce the rational expression by dividing out a common factor, called **cancelling factors**, and show that $\frac{2x+2}{x^2+3x+2}$ can be reduced to $\frac{2}{x+2}$.

* **Complete the factoring and simplification:**

$$\frac{2x+2}{x^2+3x+2} = \frac{2(\cancel{x+1})^{1}}{(x+2)(\cancel{x+1})_{1}} = \frac{2}{x+2}$$

- When $\frac{2x+2}{x^2+3x+2}$ is written in the form $\frac{2}{x+2}$, it is said to be in **lowest terms** or **simplest form**.

- $\frac{2}{x+2}$ cannot be further reduced by cancelling terms. **The two 2s cannot** be reduced.

  i.e. $\frac{2}{x+2}$ is **NOT** equivalent to $\frac{1}{x+1}$ (Replace x by any permissible value to verify this.)

- **To reduce fractions we cancel factors, not terms.**

Investigating Nonpermissible Values

a) Complete the table.

Write the value as **not defined** if the value cannot be calculated.

Value of x	Value of $\frac{2x+2}{x^2+3x+2}$	Value of $\frac{2}{x+2}$
0	1	1
–1	$\frac{0}{0}$ = undefined	2
–2	$-\frac{2}{0}$ = undefined	$\frac{2}{0}$ = undefined
–3	–2	–2

b) For which value(s) of x is the expression $\frac{2x+2}{x^2+3x+2}$ not defined? $x = -1, -2$

c) For which value(s) of x is the expression $\frac{2}{x+2}$ not defined? $x = -2$

d) Why do the values in b) and c) result in the expressions not being defined?

* cannot divide by zero *

Values of the variable which result in the value of a rational expression not being defined are called **nonpermissible values.** (aka restrictions)

Nonpermissible Values and Restrictions

Nonpermissible values are

- known as the **restrictions** on the variable.
- values of the variable which **make the denominator equal to zero**.

Note that although $\frac{2x+2}{x^2+3x+2}$ and $\frac{2}{x+2}$ are equivalent forms of a rational expression, they have different restrictions on the value of x.

Factor
Restrictions
Simplify

- The restrictions must be determined before dividing out common factors.
- Be aware that nonpermissible values are present each time we divide by an expression containing a variable.

A rational expression is in **simplest form** if its numerator and denominator have **no** common factor other than 1.

Class Ex. #1

Express in simplest form, stating the nonpermissible values of the variable.

a) $\dfrac{\overset{6}{\cancel{12x^2}}}{\underset{1}{\cancel{2x}}}$ b) $\dfrac{\overset{1}{\cancel{(a+1)}}(a-6)}{(a+7)\underset{1}{\cancel{(a+1)}}}$ c) $\dfrac{y+4}{y^2-y-20}$ d) $\dfrac{x^2+11x+28}{x^2-49}$

$x \neq 0$

$\boxed{6x}$

$a \neq -7, -1$

$\boxed{\dfrac{a-6}{a+7}}$

$\dfrac{\overset{1}{\cancel{y+4}}}{(y-5)\underset{1}{\cancel{(y+4)}}}$

$y \neq 5, -4$

$\boxed{\dfrac{1}{y-5}}$

$\dfrac{\overset{1}{\cancel{(x+7)}}(x+4)}{\underset{1}{\cancel{(x+7)}}(x-7)}$

$x \neq \pm 7$

$\boxed{\dfrac{x+4}{x-7}}$

Complete Assignment Questions #1 - #8

Assignment

$\boxed{\#1\text{-}8\ (a, ce\ldots)}$

1. Determine the nonpermissible values of the variable.

a) $\dfrac{6}{8x-7}$ b) $\dfrac{y}{10y+20}$ c) $\dfrac{5a}{5-a}$ d) $\dfrac{a^2+7a+12}{(a+4)(a+5)}$ e) $\dfrac{12y^2-2}{y}$

f) $\dfrac{1+16x^2}{1-16x^2}$ g) $\dfrac{40p^3-4}{8q^3}$ h) $\dfrac{3}{x^2+13x+12}$ i) $\dfrac{d}{d^2-8d+16}$

2. Express in simplest form, stating the nonpermissible values of the variable.

a) $\dfrac{4ab}{16a}$ b) $\dfrac{25x^3y^4}{5y^9}$ c) $\dfrac{(a+3)(a-8)}{(a+1)(a-8)}$ d) $\dfrac{(x+7)(x-2)}{x(x-2)(x+14)}$