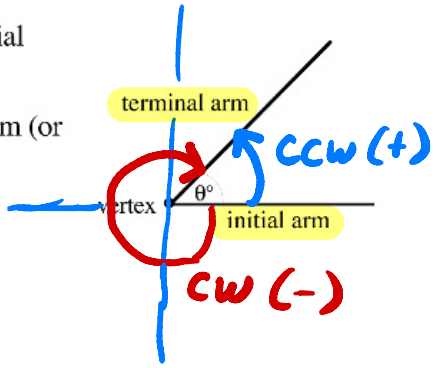
# Trigonometry - Angles and Ratios Lesson #1:
## Rotation Angles and Reference Angles

### *Angles in Standard Position*

Angles can be measured in degrees where 360° is one complete rotation.

A ***rotation angle*** is formed by rotating an initial arm (or initial side) through an angle $\theta$° about a fixed point (the vertex).

The angle formed between the initial arm and the terminal arm (or terminal side) is the rotation angle.




CCW

A ***positive angle*** results from a counter clockwise rotation.

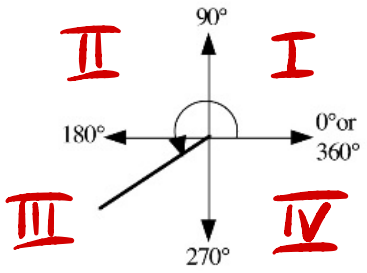
CW

A ***negative angle*** results from a clockwise rotation.

The angle shown in the above diagram is said to be in ***standard position***.

On a coordinate grid, standard position means the initial arm is along the positive $x$-axis and the rotation is about the origin.

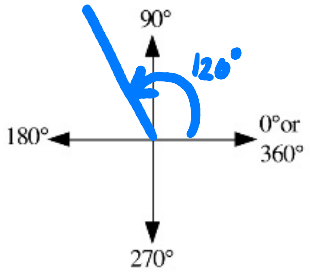
The diagram below shows an angle of 220° in standard position.

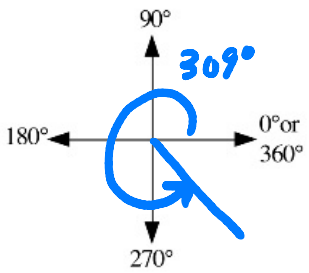


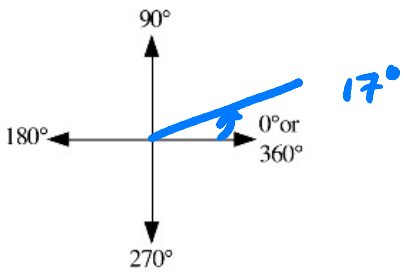

---

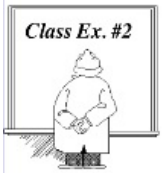

Sketch the rotation angle in standard position and state the quadrant in which the angle terminates.

**a)** 120° II **b)** 309° IV **c)** 17° I



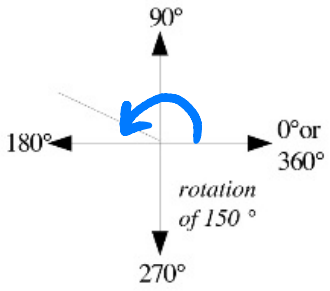



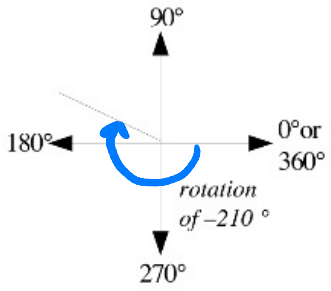



Draw the rotation angle in standard position.

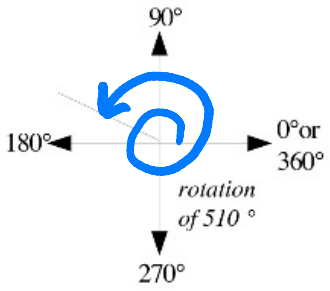
**a)**




**b)**




**c)**




Angles with the same terminal arm are called ***coterminal angles***.

Since 150° is the measure of the smallest positive rotation angle coterminal with the angles in Class Example #2, it is called the ***principal angle***.

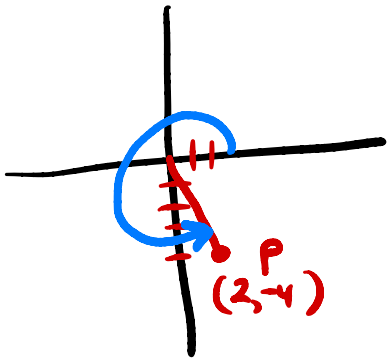
The principal angle will always have a measure between 0° and 360°.

There are infinitely many angles that are coterminal with a given angle.

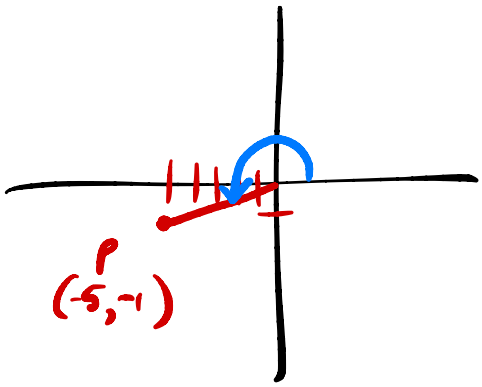
coterminals are found by: $\theta + 360°n$ where $n$ is an integer


The point $P$ lies on the terminal arm of the angle $\theta°$. Draw the angle $\theta°$ in standard position.

**a)** $P(2, -4)$




**b)** $P(-5, -1)$




**Complete Assignment Question #1 - #2**

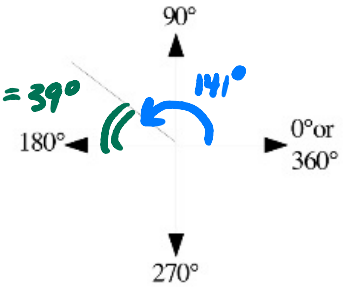
## *Reference Angles*

always true.

A **reference angle** is the acute angle formed between the terminal arm of the rotation angle and the $x$-axis.

The diagram shows the terminal arm of a rotation angle of 141° with a reference angle of 39°.

Mark 141° and 39° on the diagram.

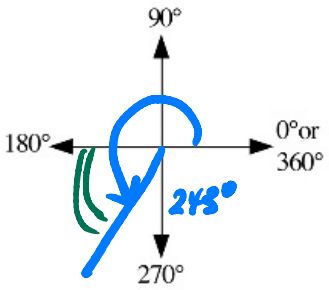



---

*Class Ex. #4*

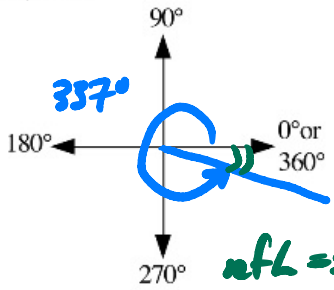
In each case, sketch the rotation angle and state the reference angle.

**a)** 243°



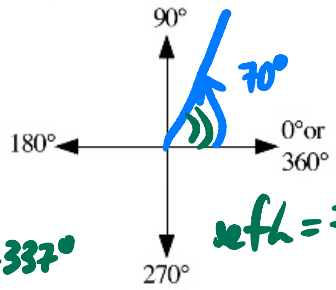

refL = 243° − 180° = 63°

**b)** 337°




refL = 360° − 337° = 23°

**c)** 70°



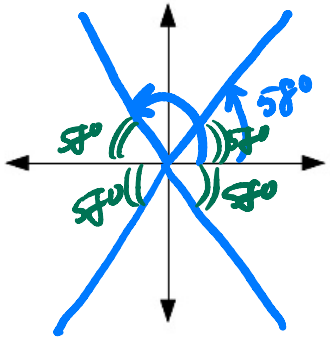

refL = 70°

---

*Class Ex. #5*

**a)** On the grid, draw a reference angle of 58° in each of quadrants one to four.

**b)** State the measure of the rotation angle in each quadrant.




I = 58°  II = 180° − 58° = 122°

III = 180° + 58° = 238°  IV = 360° − 58° = 302°

**c)** Let $P(5, 8)$ be a point on the terminal arm of the rotation angle in quadrant one. State the coordinates of points $Q$, $R$, and $S$ which are on the terminal arms of the rotation angles in quadrant two, quadrant three, and quadrant four, respectively.


Determine the measure of the rotation angle, $x$, $0° \le x < 360°$, given the reference angle and the quadrant.

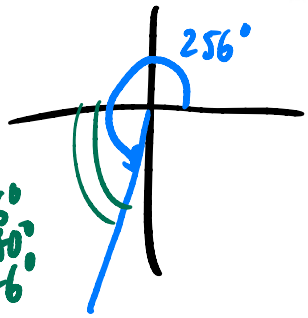
| Reference Angle | Quadrant | Sketch | Rotation Angle |
|---|---|---|---|
| 25° | 2 | | |
| 60° | 4 | | |
| 8° | 3 | | |
| 39° | 1 | | |
| 90° | between 3 and 4 | | |


Determine three angles between 0° and 360° which have the same reference angle as a rotation angle of 256°.

I → 76°

II → 180° − 76° = 104°

IV → 360° − 76° = 284°



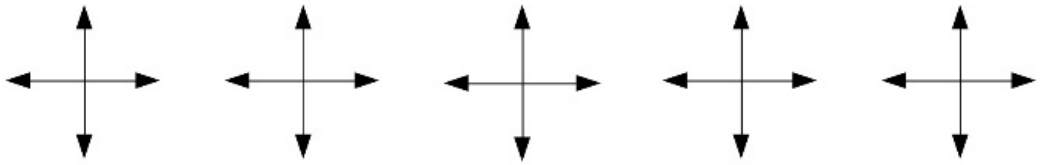

ref ∠ = 256° − 180° = 76°

**Complete Assignment Question #3 - #19**

## Assignment #1–10, 14

**1.** Sketch the following rotation angles in standard position, and state the quadrant in which the angle terminates.

**a)** 135° **b)** 300° **c)** 190° **d)** 70° **e)** 270°



**2.** In each case, the given point is on the terminal arm of an angle of $\theta°$. Draw the angle $\theta°$ in standard position.

**a)** $P(7, -4)$ **b)** $Q(-2, 3)$ **c)** $R(-1, -4)$