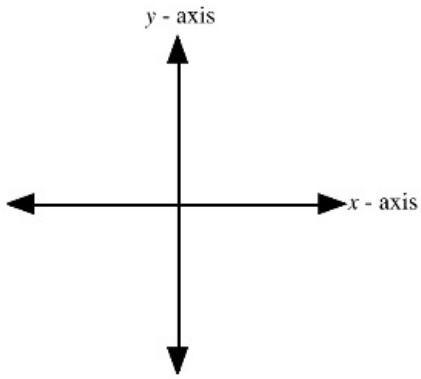
Exact Values of Trigonometric Ratios Given a Point on a Terminal Arm

In the previous lesson, we were able to determine the exact values of the trigonometric ratios given a point on the terminal arm of a rotation angle in quadrant one.
In this lesson, we extend the method into quadrants two to four.

The point $P(-3, 2)$ lies on the terminal arm of an angle θ in standard position. Complete the following procedure to determine the values of the primary trigonometric ratios.

a) Sketch the rotation angle on the grid and mark the point $P(-3, 2)$ on the terminal arm.

b) Calculate the exact length of $OP = r$.



c) Use $x = -3$, $y = 2$ and r from above to write the three trigonometric ratios for angle θ.

The point $(-4, -2)$ lies on the terminal arm of an angle θ in standard position. Determine the exact value of $\sin\theta$.

$x = -4$
$y = -2$

$\sin\theta = \frac{y}{r}$

$r^2 = x^2 + y^2$

$r = \sqrt{(-4)^2 + (-2)^2}$

$r = \sqrt{20} = 2\sqrt{5}$

$\sin\theta = -\frac{2}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{10} = -\frac{\sqrt{5}}{5}$

Complete Assignment Questions #1 - #3

Value of a Trigonometric Ratio Given a Different Trigonometric Ratio

Angle A terminates in the third quadrant with $\sin A = -\frac{4}{5}$. Complete the following procedure to determine the values of $\cos A$ and $\tan A$.

a) Since $\sin A = -\frac{4}{5} = \frac{y}{r}$, we know that the point $(x, -4)$ lies on the terminal arm in the third quadrant with $r = 5$. Sketch a diagram, draw the reference triangle and mark x, $y = -4$, and $r = 5$ on the reference triangle.

b) Use $x^2 + y^2 = r^2$ to determine the value of x. (Note that in quadrant three, the value of x must be negative).

c) Use the values of x, y, and r to determine the exact values of $\cos A$ and $\tan A$.

If $\tan\theta = -\frac{2}{3}$ and $\cos\theta$ is positive, then find the exact value of $\sin\theta$.

IV

$x = 3$

$y = -2$

$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{-2\sqrt{13}}{13}}$

$r = \sqrt{3^2 + (-2)^2}$

$r = \sqrt{13}$

Complete Assignment Questions #4 - #11

#1-11