

# Trigonometry - Angles and Ratios Lesson #4: Solving Simple Trigonometric Equations

## Solving Trigonometric Equations with the Domain $0^\circ \leq \theta \leq 360^\circ$

We can use the concepts of reference angles and signs of the trigonometric ratio to solve equations of the form  $\sin \theta = a$ ,  $\cos \theta = a$ , or  $\tan \theta = a$ , where  $0^\circ \leq \theta \leq 360^\circ$ .

Use the following procedure to solve an equation such as  $\sin \theta = 0.5$ , where  $0^\circ \leq \theta \leq 360^\circ$ .

**Step 1:** Determine the **quadrant(s)** the angle will be in by looking at the sign of the ratio.

**Step 2:** Determine the reference angle (always between  $0^\circ$  and  $90^\circ$ ) and draw a rough sketch in the appropriate quadrant(s). To determine the reference angle, use

**2nd** **sin** or **2nd** **cos** or **2nd** **tan** *ref L*  
of the **absolute value** of the given quantity.

**Step 3:** Determine the rotation angle(s) using the reference angle and the quadrant(s).



- Always check the given domain to determine which quadrants are valid in the calculation. Sometimes the domain is restricted to, for example,  $0^\circ \leq \theta \leq 180^\circ$ , or  $90^\circ \leq \theta \leq 180^\circ$ .

*restrictions on the answers.*



Use the procedure above to solve  $\sin \theta = 0.5$ , where  $0^\circ \leq \theta \leq 360^\circ$

*sin* ↑

$\sin^{-1}(0.5) = 30^\circ$     **I**  $30^\circ$     **II**  $150^\circ$

*ref L = 30°*



Find the measure of  $x$ , to the nearest degree, where  $0^\circ \leq x \leq 360^\circ$ .

a)  $\sin x = -0.8090$

b)  $\cos x = -0.8090$

c)  $\tan x = -2.4586$

*III* ↑ *IV*    *II* ↑ *III*    *II* ↑ *IV*

$\sin^{-1}(-0.8090) = 54^\circ$   
*ref L*

$\cos^{-1}(-0.8090) = 144^\circ$   
*ref L = 36°*

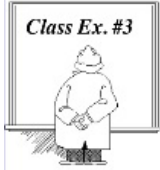
$\tan^{-1}(-2.4586) = 68^\circ$   
*ref L = 68°*

**III**  $234^\circ$     **IV**  $306^\circ$

**II**  $144^\circ$     **III**  $216^\circ$

**II**  $112^\circ$     **IV**  $292^\circ$

Class Ex. #3



Solve the following equations if  $0^\circ \leq \theta \leq 360^\circ$ .

a)  $\sin \theta = 1$

b)  $\cos \theta = 0$

Class Ex. #4



Solve the equation  $3 \tan \theta + 1 = 4$ ,  $0^\circ \leq \theta \leq 180^\circ$ .

$$\frac{3 \tan \theta}{3} = \frac{3}{3}$$

$$\tan \theta = 1$$

$$\tan^{-1}(1) = 45^\circ$$

refl =  $45^\circ$

$$I = \boxed{45^\circ}$$

Complete Assignment Questions #1 - #4

***Solving Trigonometric Equations Outside the Domain  $0^\circ \leq \theta \leq 360^\circ$***

Use the following procedure to solve equations where the domain is outside of  $0^\circ \leq \theta \leq 360^\circ$ .

Step 1: Solve the equation with the domain  $0^\circ \leq \theta \leq 360^\circ$  using the steps on the previous page.

Step 2: Using the concepts of coterminal angles, add or subtract  $360^\circ$  or multiples of  $360^\circ$ .

Class Ex. #5



Solve the equation  $\sqrt{3} \tan \theta + 1 = 0$ ,  $0^\circ \leq \theta \leq 720^\circ$  \*

$$\frac{\sqrt{3} \tan \theta = -1}{\sqrt{3}} \quad \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan \theta = -\frac{1}{\sqrt{3}}$$

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -30^\circ$$

ref L =  $30^\circ$

$\overset{\text{II}}{\boxed{150^\circ}}$   
 $+ 360^\circ$   
 $\downarrow$   
 $\boxed{510^\circ}$

$\overset{\text{IV}}{\boxed{330^\circ}}$   
 $+ 360^\circ$   
 $\downarrow$   
 $\boxed{690^\circ}$

Class Ex. #6



To the nearest whole number, solve the equation  $\cos x = -0.82$  where  $-360^\circ \leq x \leq 0^\circ$ .

# 1-7 (a, c, e...)

Complete Assignment Questions #5 - #9

## Assignment

1. Solve the following equations, where  $0^\circ \leq \theta \leq 360^\circ$ .

a)  $\cos \theta = \frac{1}{2}$

b)  $\sin \theta = -\frac{\sqrt{3}}{2}$

c)  $\tan \theta = -1$