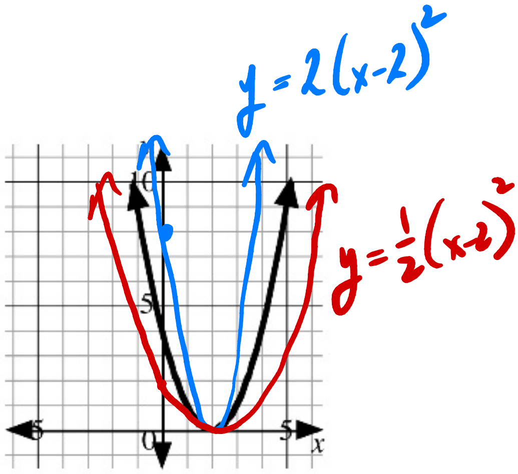
Quadratic Functions and Equations Lesson #3:
Analyzing Quadratic Functions - Part Two

In the last lesson we analyzed the graph of $y = (x-p)^2 + q$ and discovered transformations associated with the parameters p and q. In this lesson we investigate the effect of the parameter, a, on the graph of $y = a(x-p)^2 + q$. The following investigations can be completed as a class lesson or as an individual assignment.

Analyzing the Graph of $y = a(x-p)^2,\ a > 0$



The graph of $y = f(x) = (x-2)^2$ is shown.

a) Write an equation which represents each of the following:

- $y = 2f(x)$ $y=2(x-2)^2$
- $y = \frac{1}{2}f(x)$ $y=\frac{1}{2}(x-2)^2$

b) Use a graphing calculator to sketch $y = 2f(x)$ and $y = \frac{1}{2}f(x)$ on the grid.

c) Complete the following by circling the correct choice and filling in the blank.

- Compared to the graph of $y = f(x)$, the number 2 in the graph of $y = 2f(x)$ results in a vertical *expansion* / *compression* by a factor of 2.
- The y intercept of the graph of $y = 2f(x)$ is double the y-intercept of the graph of $y = f(x)$.

d) Complete the following by circling the correct choice and filling in the blank.

- Compared to the graph of $y = f(x)$, the number $\frac{1}{2}$ in the graph of $y = \frac{1}{2}f(x)$ results in a vertical *expansion* / *compression* by a factor of $\frac{1}{2}$.
- The y intercept of the graph of $y = \frac{1}{2}f(x)$ is half the y-intercept of the graph of $y = f(x)$.

- In mathematics, the general name given to an expansion or a compression is a **stretch**.
- A vertical stretch is "anchored" by the x-axis, i.e. the x-coordinate of every point on the original graph will not change and the y-coordinate of every point is multiplied by a factor of a.
- In some texts a compression is called a contraction.

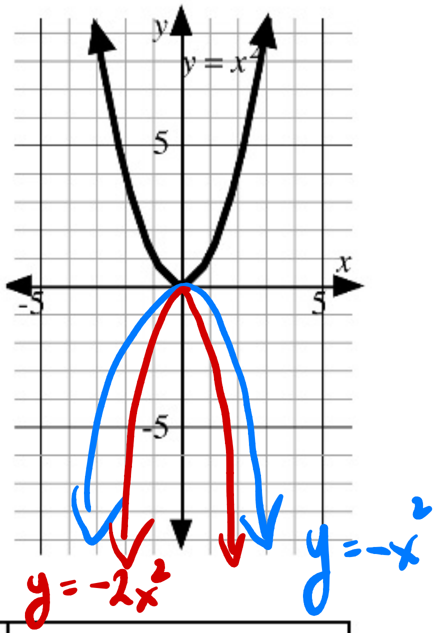
e) Describe the effect of the **parameter**, a, on the graph of $y = a(x-p)^2$ where $a > 0$.

vertical expansion or compression by a factor of "a"

f) Compared to the graph of $y = x^2$, the graph of $y = ax^2$ results in a vertical stretch of factor a about the x-axis.
If $a > 1$, the parabola undergoes a vertical expansion.
If $0 < a < 1$, the parabola undergoes a vertical compression.

Analyzing the Graph of $y = ax^2$, $a < 0$

The graph of $y = f(x) = x^2$ is shown.



a) Write an equation which represents

- $y = -f(x)$ $y = -x^2$
- $y = -2f(x)$ $y = -2x^2$

b) Use a graphing calculator to sketch $y = -f(x)$ and $y = -2f(x)$.

c) Complete the following chart. The first row is done.

Function	*Equation Representing Function*	*Vertex*	*Max/Min Value*	*Equation of Axis of Symmetry*	*Description of Transformation*
$y = f(x)$	$y = x^2$	(0, 0)	min, 0	$x = 0$	no transformation
$y = -f(x)$	$y = -x^2$	(0,0)	max, 0	$x = 0$	reflection on the x-axis
$y = -2f(x)$	$y = -2x^2$	(0,0)	max, 0	$x = 0$	reflection on the x-axis vert. exp. by a factor of 2
$y = af(x)$, where $a < 0$	$y = ax^2$	(0,0)	max, 0	$x = 0$	reflection on the x-axis vert stretch by a factor of $\|a\|$

d) How does the graph of $y = -x^2$ compare to the graph of $y = x^2$?

reflection on the x-axis (direction of opening is now down)

e) Compared to the graph of $y = x^2$, the graph of $y = ax^2$, $a < 0$ results in a reflection in the x-axis and a vertical stretch by a factor of $|a|$ about the x-axis.

Transformations Associated with the Parameters of $y = a(x - p)^2 + q$

Compared to the graph of $y = x^2$, the following transformations are associated with the parameters of $y = a(x - p)^2 + q$.

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a indicates a vertical stretch about the x-axis.

- If $a > 1$ there is an expansion.
- If $0 < a < 1$ there is a compression.
- If $a < 0$, there is also a reflection in the x-axis.

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p indicates a horizontal translation.
- If $p > 0$, the parabola moves p units right.
- If $p < 0$, the parabola moves p units left.

q indicates a vertical translation.
- If $q > 0$, the parabola moves q units up.
- If $q < 0$, the parabola moves q units down.

(p, q) are the coordinates of the vertex.

$x = p$ is the equation of the axis of symmetry.

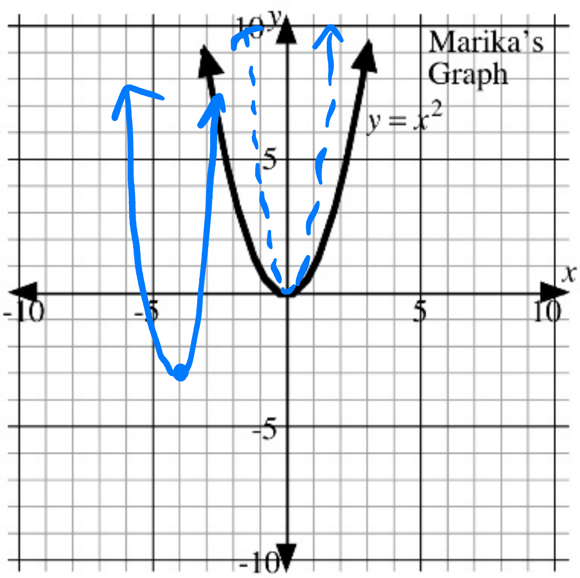
Consider the function $f(x) = 2(x + 4)^2 - 3$.

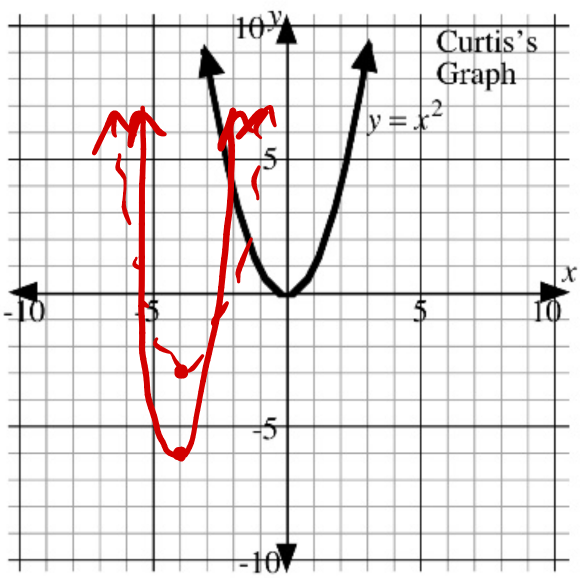
a) State the transformations applied to the graph of $y = x^2$ which would result in the graph of $y = 2(x + 4)^2 - 3$.

① vert. exp. by a factor of 2
② hor. trans. 4 units left
③ vert. trans. 3 units down.

b) Marika and Curtis were discussing how to graph this function without using a graphing calculator. Marika suggested doing the stretch followed by the translation. Curtis suggested doing the translation followed by the stretch.

- Complete the grids below to show the graphs obtained by each student.
- Use a graphing calculator to determine which student is correct.





Unless otherwise indicated, use the following order to describe how to transform from one graph to another.

1. Stretches
2. Reflections
3. Translations

Describe how the graphs of the following functions relate to the graph of $y = x^2$.

a) $y = -\frac{1}{4}x^2$

① refl. on the x-axis
② vert. comp. by a factor of $\frac{1}{4}$

b) 3. 2. $\left[\frac{1}{3}y = (x+6)^2\right]$

$y = 3(x+6)^2$

① vert. exp. by a factor of 3
② hor. trans. 6 units left

The following three transformations are applied, in order, to the graph of $y = x^2$: a reflection in the x-axis, a vertical stretch by a factor of $\frac{1}{3}$ about the x-axis, and a translation 7 units right. At the end of the three transformations, the point $(1, t)$ is on the resulting graph.

a) Find the equation of the image function after each transformation.

$y = x^2 \longrightarrow y = -x^2 \longrightarrow y = -\frac{1}{3}x^2 \longrightarrow y = -\frac{1}{3}(x-7)^2$

b) State the coordinates of the vertex of the final graph. $(7, 0)$

c) Find the value of t.

let $x = 1$

$\boxed{t = -12}$

Complete the following table.

Function	Vertex	Max/Min Value	Equation of Axis of Symmetry	Domain	Range
$y = -(x+3)^2 - 4$	$(-3,-4)$	max, -4	$x = -3$	$x \in \mathbb{R}$	$y \leq -4$
$y = 3(x-9)^2$	$(9,0)$	min, 0	$x = 9$	$x \in \mathbb{R}$	$y \geq 0$

Complete Assignment Questions #1 - #10