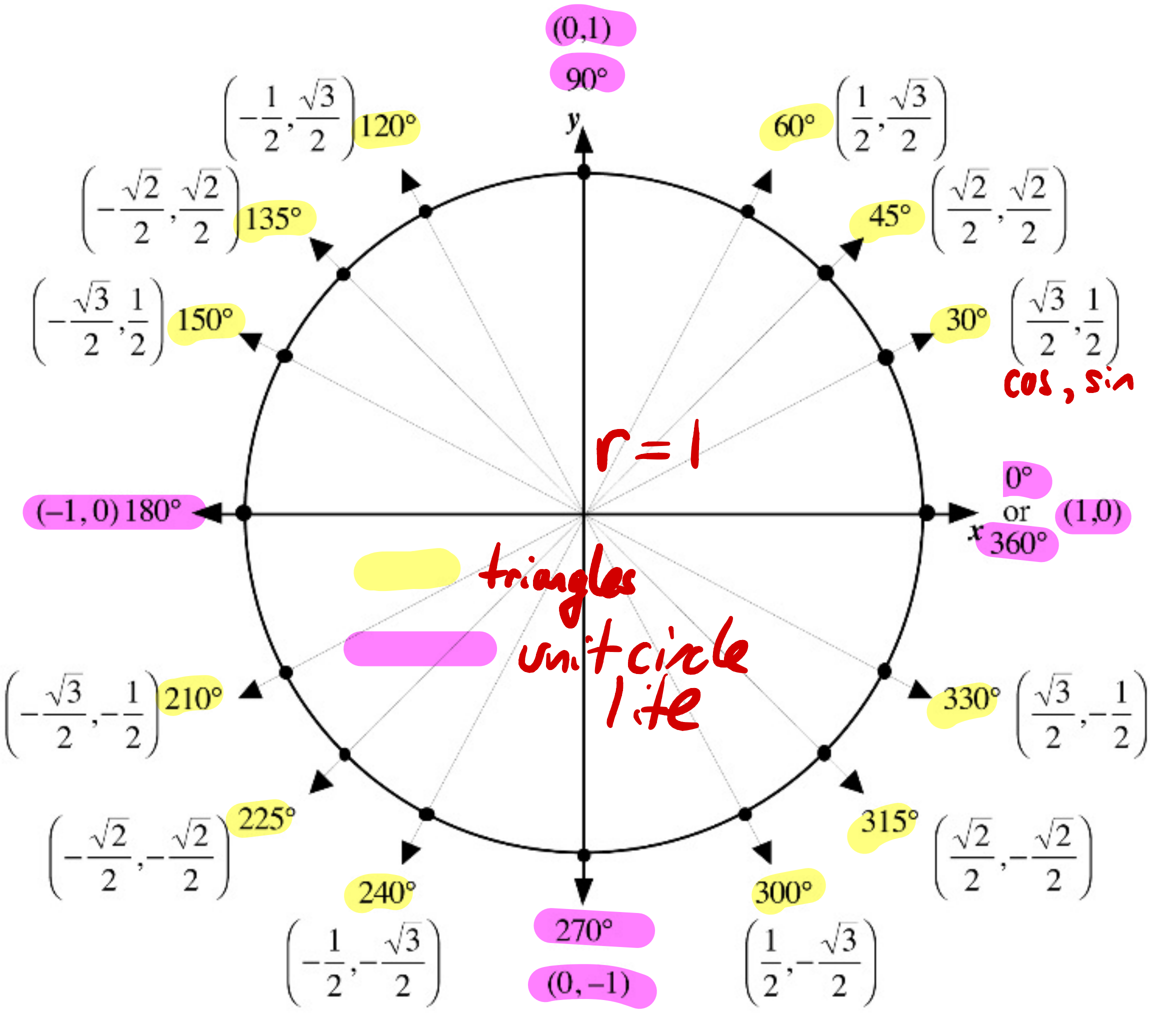
The Unit Circle

The unit circle can be formed by reflecting the above diagram in the x-axis, in the y-axis, and in both the x-axis and the y-axis.



The circle above, with a radius of one unit, is called the **unit circle** and it is important to understand how it works.

Recall the formulas $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$, and $\cot\theta = \frac{x}{y}$.

* 1, unit circle! 1

- In the unit circle, where $r = 1$, we have

  $\sin\theta$ = y   and $\cos\theta$ = x

(x, y)
(cos, sin)

- Every point on the unit circle has coordinates (x, y) which can be written as $(\cos\theta, \sin\theta)$.

- $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Class Ex. #1

Use the unit circle to find the exact value of the trigonometric ratios for a rotation angle of 240°. Give each answer with a rational denominator.

sin 240° = cos 240° = tan 240° =

Class Ex. #2

Use the unit circle to find the exact value of no calculator

a) cos 135° **b)** tan 120° **c)** sin 180° **d)** tan 270°

a) ref ∠ = 45°, II; $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}/2$

b) ref ∠ = 60°, II; $= -\sqrt{3}$

c) ref ∠ = 0°, (−1, 0); sin 180° = 0

d) ref ∠ = 90°, (0, −1); tan 270° = ∅ undefined

Class Ex. #3

cos A, sin A   cos B, sin B

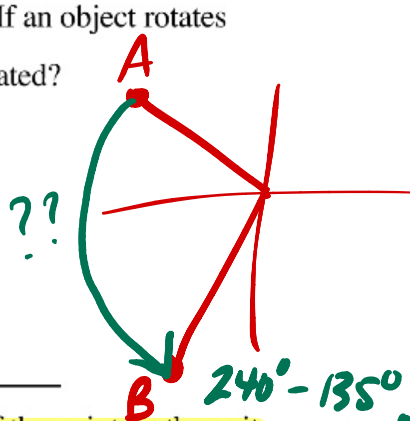
$A\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $B\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ are two points on the unit circle. If an object rotates counterclockwise from point A to point B, through what angle has it rotated?

r = 1

∠A → II   ∠B → III

ref ∠ = 45°   ref ∠ = 60°

∠A = 135°   ∠B = 240°



240° − 135° = 105°

Class Ex. #4

Use a calculator to determine, to four decimal places, the coordinates of the point on the unit circle that corresponds to a rotation of 148°.

(x, y) → (cos 148°, sin 148°)

→ (−0.8480, 0.5299)

Class Ex. #5

(cos, sin)

The point $T(-0.8829, 0.4695)$ lies on the unit circle. Determine the value of θ, where θ is the angle made by the positive x-axis and the line passing though T.

$\cos^{-1}(-0.8829) = \boxed{152°}$

II

Note

We now have two methods for determining exact values of trigonometric ratios of certain angles greater than 90°. Use either method.

Class Ex. #6

Use the chart or unit circle to find the exact value of

a) $\cos 300° + \sin 330°$ **b)** $\sin^2 225° + \cos^2 225°$ **c)** $\dfrac{2\tan 150°}{1-\tan^2 150°}$

ref∠ = 60° in IV ref∠ = 30° in IV

$= \frac{1}{2} + \left(-\frac{1}{2}\right)$

$= \boxed{0}$

ref∠ = 45° in III

$= \left(-\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2$

$= \frac{1}{2} + \frac{1}{2}$

$= \boxed{1}$

ref∠ = 30° in II (−)

$= \dfrac{2\left(-\frac{\sqrt{3}}{3}\right)}{1-\left(-\frac{\sqrt{3}}{3}\right)^2}$

$= \dfrac{-\frac{2\sqrt{3}}{3}}{1-\frac{1}{3}}$

$= -\frac{2\sqrt{3}}{3} \cdot \frac{3}{2} = \boxed{-\sqrt{3}}$

$\sin^2\theta = (\sin\theta)^2$

Complete Assignment Questions #2 - #13

#2-13 (a, c, e...)

Assignment

1. The diagram on page 201 has been reflected in the x-axis, the y-axis, and in both axes to produce the diagram below. Complete the diagram by writing the coordinates and the rotation angle for each point on the circumference of the circle.