Trigonometry - Sine and Cosine Laws Lesson #2: The Sine Law

Trigonometry in Acute Angled and Obtuse Angled Triangles

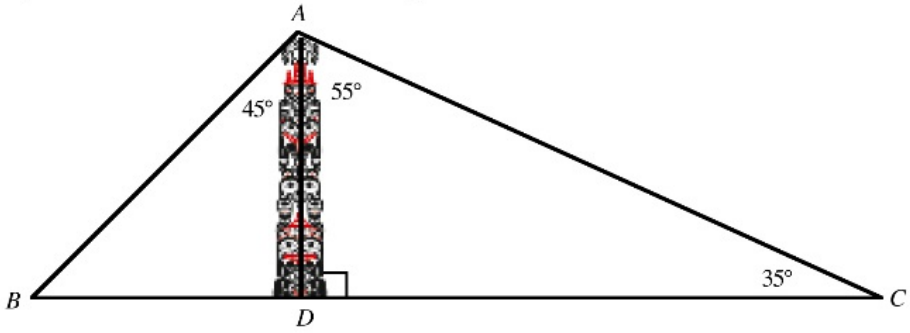
In the last lesson, we reviewed trigonometry in right triangles using SOHCAHTOA.

In the next three lessons, we focus on solving triangles which are not right angled and in which SOHCAHTOA is not valid. nor is Pythagorus.

In the next section of work we will determine the side of an acute angled triangle by
i) splitting it in two right triangles and using SOHCAHTOA as in Class Ex. #1
ii) using the Sine Law as in Class Ex. #2

To honor the 20th anniversary of the Gwaii Haanas Agreement, more than 400 people in 2013 participated in the monumental raising of a 42 foot Gwaii Haanas Legacy Pole. The poles design was inspired by the connections between the Haida Nation and all those who care for Gwaii Haana from mountain-top to seafloor. The pole tells the story of a ground breaking cooperative agreement between the Haida Nation and the Government of Canada to protect Gwaii Haanas.

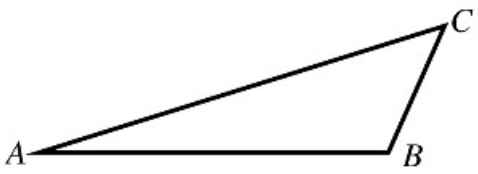
Triangle *ABC* has three acute angles that represent the possible angles used when the pole is standing at 90 degrees to the ground. Master Carver, Jaalen Edenshaw and two assistants, will need to determine both the ground clearance needed and the rope required to raise the pole successfully. Use SOHCAHTOA to determine both the ground clearance (length of *BC*) and the length of rope required when the pole is standing vertical. Work to three decimal places and answer to two decimal places.



A New Notation

Often, in trigonometry, it is convenient to use the following notation.

In triangle ABC, represent
the length of the side opposite angle A by a,
the length of the side opposite angle B by b,
and the length of the side opposite angle C by c.

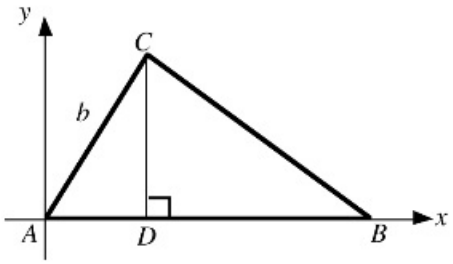


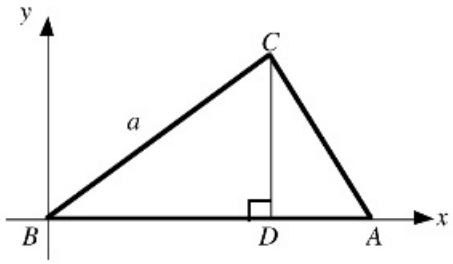
The Sine Law

In every triangle ABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ or $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Proof of the Sine Law

The diagrams show the same triangle ABC placed with base AB on the x–axis.
In diagram i) the origin is at A, and in diagram ii) the origin is at B.
The line CD is drawn perpendicular to AB.





1. Complete the following work to show that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$.

- In i) $\sin A = \dfrac{CD}{AC} = \dfrac{CD}{b}$
- In ii) $\sin B =$ $=$

$CD =$ $CD =$

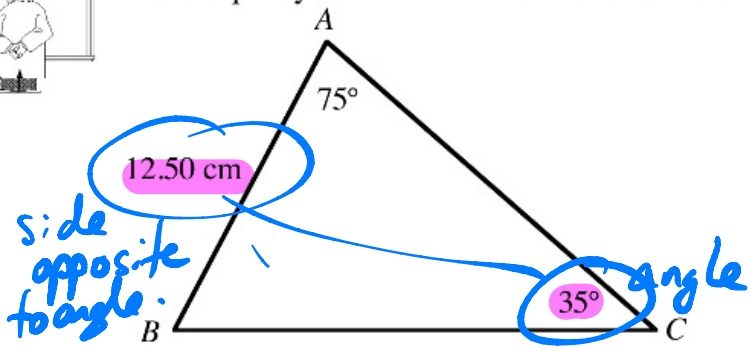
- It follows that $b \sin A$ $=$
- Dividing both sides by $\sin A \sin B$ gives the result _____ .

2. Repeating the work above with AC placed on the x–axis would give the result $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$.

Hence $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ or $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

To use the sine law, we need to know **three** pieces of information.
This information must include both numerator and denominator of one of the three fractions, i.e. we need to know an angle and the measure of its opposite side.

Triangle *ABC* from Class Ex. #1 is shown. Use the sine law to calculate the length of *BC*, and compare your answer to the SOHCAHTOA method.



side opposite to angle

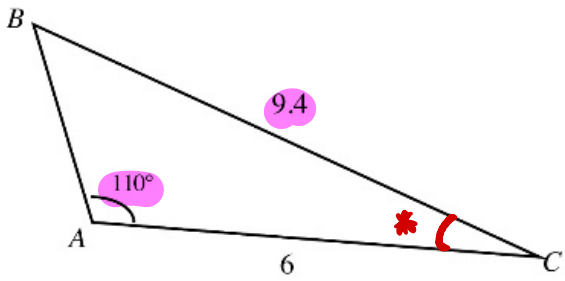
angle

$\frac{\sin 35^\circ}{12.50} = \frac{\sin 75^\circ}{BC}$

$\frac{BC \sin 35^\circ}{\sin 35^\circ} = \frac{12.5 \sin 75^\circ}{\sin 35^\circ}$

$BC = 21.05 \text{ cm}$.

Use the sine law in the triangle shown to determine the measure of $\angle ACB$ to the nearest degree.



$\frac{\sin 110^\circ}{9.4} = \frac{\sin B}{6}$

$\frac{6 \sin 110^\circ}{9.4} = \frac{9.4 \sin B}{9.4}$

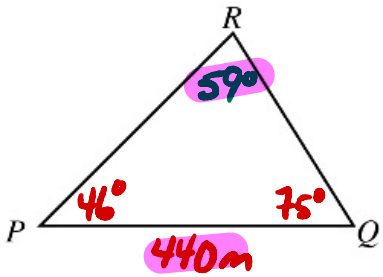
$ANS = \sin B$

$\sin^{-1}(ANS) = \angle B = 37^\circ$

$\angle C = 180^\circ - (110^\circ + 37^\circ) = 33^\circ$

A surveyor measures a base line *PQ* to be 440 m long. He takes measurements of a landmark *R* from *P* and *Q*, and finds that $\angle QPR = 46^\circ$ and $\angle PQR = 75^\circ$.

a) Calculate the perimeter of ΔPQR to the nearest metre.



$\angle R = 180^\circ - (46^\circ + 75^\circ)$

$= 59^\circ$

$\frac{\sin 59^\circ}{440} = \frac{\sin 46^\circ}{RQ}$ $\qquad$ $\frac{\sin 59^\circ}{440} = \frac{\sin 75^\circ}{PR}$

$RQ = \frac{440 \sin 46^\circ}{\sin 59^\circ}$ $\qquad$ $PR = \frac{440^\circ \sin 75^\circ}{\sin 59^\circ}$

$= 369.25 \text{ m}$ $\qquad$ $= 495.83$

Perimeter $= 440 + 369.25 + 495.83 = 1305 \text{ m}$

 # 1-4, 7-9