Quadratic Functions and Equations Lesson #5:
Converting from General Form to Standard Form by Completing the Square

Review

CTS Completing the Square

- The **general form** of a quadratic function has the equation $y = ax^2 + bx + c$.
- The **standard form** of a quadratic function has the equation $y = a(x-p)^2 + q$.
- Writing a function in standard form enables us to analyze the function more easily e.g. we can determine the vertex, axis of symmetry and maximum / minimum value of the function.

Completing the Square

$(x+4)^2$ and $(x-5)^2$ are examples of **perfect squares.**

a) Expand the following perfect squares.

$(x+4)^2 = (x+4)(x+4) =$ $(x+4)^2$ # $\qquad (x+7)^2 = (x+7)(x+7) =$ ____________

$(x-5)^2 = (x-5)(x-5) =$ ____________ $\qquad (x-1)^2 = (x-1)(x-1) =$ ____________

$(x+a)^2 =$ ____________ $\qquad (x-a)^2 =$ ____________

b) Factor the following expressions into perfect squares.

$x^2 + 6x + 9 =$ ____________ $\qquad x^2 + 12x + 36 =$ ____________

$x^2 - 4x + 4 =$ ____________ $\qquad x^2 - 16x + 64 =$ ____________

c) Add an appropriate constant so that the following expressions can be written as perfect squares.

$x^2 + 2x +$ __ $=$ _______ $\qquad x^2 + 18x +$ __ $=$ _______

$x^2 - 3x +$ __ $=$ _______ $\qquad x^2 - \frac{1}{4}x +$ __ $=$ _______

The process of adding a constant term to a quadratic expression to make it a perfect square is called **completing the square**.

To complete the square of $x^2 + bx$, add $\left(\frac{1}{2}\text{coefficient of } x\right)^2$

i.e. add $\left(\frac{1}{2}b\right)^2$ to give $\left(x + \frac{1}{2}b\right)^2$.

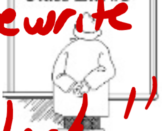
Writing $f(x) = x^2 + bx + c$ in Standard Form by Completing the Square

1.) isolate the constant
2.) If necessary factor "a" off the 1st 2 terms
3.) complete the square. ($\frac{1}{2}b \rightarrow$ squaring)
4.) Add into the brackets
5.) build a bridge
6.) DO-UNDO
7.) rewrite
8.) check!! the y-int.

Use the following process to convert a function of the form $f(x) = x^2 + bx + c$ into standard form.

Step 1: Take half of the numerical coefficient of the x-term and square it.
→
Step 2: Add and subtract the squared number in step 1. (This keeps the value of the function the same.)
→
Step 3: Form a perfect square, write it in factored form, and simplify.

Class Ex. #1



Express $y = x^2 + 10x + 16$ in completed square form.
Use a graphing calculator to verify that both equations are represented by identical graphs.

$$y = x^2 + 10x \qquad + 16$$

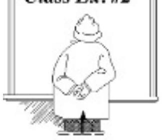
$$\frac{1}{2}(10) = \boxed{5}^2 = 25$$

$$y = \underbrace{(x^2 + 10x + 25)}_{+25} + 16 - 25$$

always

$$\boxed{y = (x+5)^2 - 9}$$

Class Ex. #2



A function, f, is defined by $f(x) = x^2 - 9x - 20$.
Determine the minimum value of f by writing the function in standard form.

$$y = x^2 - 9x - 20$$

$$\frac{1}{2}(-9) = \boxed{\frac{-9}{2}} = \frac{81}{4}$$

$$y = \underbrace{\left(x^2 - 9x + \frac{81}{4}\right)}_{+81/4} - 20 - \frac{81}{4}$$

$$\boxed{y = \left(x - \frac{9}{2}\right)^2 - \frac{161}{4}}$$

Complete Assignment Questions #1 - #4