Form a quadratic equation and solve. $\dfrac{2}{a^2} + \dfrac{3}{a} = -1, \quad a \neq 0$

Complete Assignment Questions #1 - #3

Extension: Enrichment

Investigating the Nature of the Roots of a Quadratic Equation

Insert the missing values.

Equation #1

$x^2 - 6x + 5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

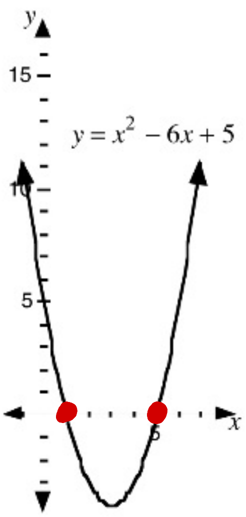
$x = \dfrac{6 \pm \sqrt{36 - 20}}{2}$

$= \dfrac{6 \pm \sqrt{16}}{2}$

$= \dfrac{6 + 4}{2}$ and $\dfrac{6 - 4}{2}$

$\therefore$ the roots are

$x = 5$ and $x = 1$



2 diff real roots

Equation #2

$x^2 - 6x + 9 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

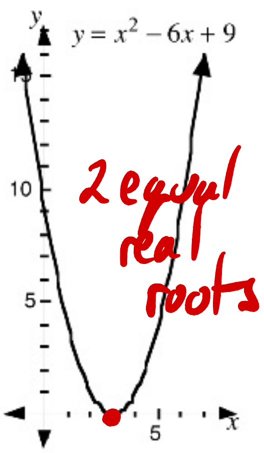
$x = \dfrac{6 \pm \sqrt{36 - 36}}{2}$

$= \dfrac{6 \pm \sqrt{0}}{2}$

$= \dfrac{6 + 0}{2}$ and $\dfrac{6 - 0}{2}$

$\therefore$ the roots are

$x = 3$ and $x = 3$



2 equal real roots

1 root

Equation #3

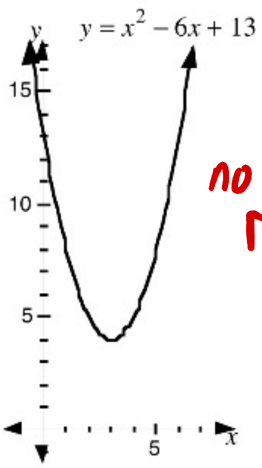
$x^2 - 6x + 13 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{6 \pm \sqrt{36 - 52}}{2}$

$= \dfrac{6 \pm \sqrt{-16}}{2}$ * $\varnothing$

$\therefore$ the roots are none



no real roots

0 roots.

The Nature of the Roots of a Quadratic Equation

The roots of a quadratic equation are represented by the x-intercepts of the graph of the corresponding quadratic function.

The roots of a quadratic equation can be **equal or unequal** and **real or non-real**.

Consider the graphs from the previous page.

- In graph 1 the roots of the equation $x^2 - 6x + 5 = 0$ are real and unequal (distinct).
- In graph 2 the roots of the equation $x^2 - 6x + 9 = 0$ are real and equal.
- In graph 3 the roots of the equation $x^2 - 6x + 13 = 0$ are non-real.

The Discriminant

The nature of the roots of a quadratic equation can be determined without actually solving the equation or drawing its graph.

The number $b^2 - 4ac$, which appears under the radical symbol in the quadratic formula can be used to discriminate between the different types of roots, and is called the **discriminant**.

discriminant $= b^2 - 4ac$ → gives us the nature of the roots.

Class Ex. #3

a) Complete the table using the calculations from the investigation on the previous page.

Equation	Roots	Nature of Roots	$b^2 - 4ac$
$x^2 - 6x + 5 = 0$	1, 5	2 diff. real roots	16
$x^2 - 6x + 9 = 0$	3, 3	1 real root	0
$x^2 - 6x + 13 = 0$		no real roots	−16

(+)

(−) no solution

b) Complete the following:

- If the discriminant $b^2 - 4ac = 0$, then the roots are ____________ and ____________ .
- If the discriminant $b^2 - 4ac > 0$, then the roots are ____________ and ____________ .
- If the discriminant $b^2 - 4ac < 0$, then the roots are ____________ .

Class Ex. #4

Determine the nature of the roots of the following equations without solving or graphing.

a) $6x^2 - x - 1 = 0$ b) $x^2 + 16 = 8x$ c) $5x^2 + 2x + 1 = 0$.

a) b^2-4ac

$(-1)^2-4(6)(-1)$

$=\boxed{25}$ ✓

2 diff. real roots. ✓

b) b^2-4ac

$(-8)^2-4(1)(16)$

$64-64=\boxed{0}$

1 real root

c) b^2-4ac

$(2)^2-4(5)(1)$

$4-20=\boxed{-16}$

no real roots.

Class Ex. #5

Determine for what value(s) of m the quadratic equation $x^2 - 8x + m$ has

a) real and distinct roots b) real and equal roots c) non-real roots

a) $b^2-4ac>0$

$(-8)^2-4(1)(m)>0$

$64-4m>0$

$\frac{64}{4} > \frac{4m}{4}$

$\boxed{16>m}$

b) $b^2-4ac=0$

$(-8)^2-4(1)(m)=0$

$64-4m=0$

$\frac{64}{4}=\frac{4m}{4}$

$\boxed{16=m}$

c) $b^2-4ac<0$

$(-8)^2-4(1)(m)<0$

$64-4m<0$

$\frac{64}{4}<\frac{4m}{4}$

$\boxed{16<m}$

Class Ex. #6

a) State a condition for $b^2 - 4ac$ so that the equation $ax^2 + bx + c = 0$ has real roots.

b) Given that the equation $ax^2 + bx + c = 0$ has real roots, state a condition for $b^2 - 4ac$ so that the roots are: **i)** rational, **ii)** irrational.

c) Show that the roots of the equation $(m - 2)x^2 - (3m - 2)x + 2m = 0$ are always real and rational.

Complete Assignment Questions #4 - #12

$\boxed{\#1, 2, 4\text{-}6}$