

# Quadratic Functions and Equations Lesson #9: Applications of Quadratic Functions - An Algebraic Approach

## Review

The standard form of quadratic functions is useful to solve, analyze, and interpret problems whose graphical model is parabolic in shape. Complete the following statements for the standard form equation of a parabola  $y = a(x - p)^2 + q$ .

- The coordinates of the vertex are \_\_\_\_\_.
- When  $a < 0$ , the maximum value is \_\_\_\_\_. When  $a > 0$ , the minimum value is \_\_\_\_\_.
- The equation of the axis of symmetry is \_\_\_\_\_.

## Maximum/Minimum Applications

In this lesson, all the questions are intended to be completed algebraically.



Consider the following information taken from Lesson 8, page 323, Class Ex. #1.

“The height,  $h$ , in metres above the ground, of a projectile at any time,  $t$ , in seconds after the launch is defined by the function  $h(t) = -4t^2 + 48t + 3$ .”

- \* a) Complete the square to write  $h$  in standard form.

$$\begin{aligned} h(t) &= -4t^2 + 48t + 3 \\ &= -4(t^2 - 12t + 36) + 3 + 144 \\ h(t) &= -4(t-6)^2 + 147 \end{aligned}$$

$$\frac{1}{2}(-12) = \boxed{-6}^2 = 36$$

- b) Find the height of the projectile 3 seconds after the launch.

let  $t = 3$   $h(3) = -4(3-6)^2 + 147$   $h(3) = \boxed{111\text{m}}$

- c) Find the maximum height reached by the projectile  $\boxed{147\text{m}}$

- d) How many seconds after the launch is the maximum height reached?  $\boxed{6\text{seconds}}$

$\boxed{3\text{m}}$

- e) What was the height of the projectile at the launch? let  $t = 0$

- f) Determine when the projectile hits the ground to the nearest tenth of a second.

$$\begin{aligned} h(t) &= 0 \\ 0 &= -4(t-6)^2 + 147 \\ \frac{-147}{-4} &= \frac{-4(t-6)^2}{-4} \\ \pm\sqrt{\frac{147}{4}} &= \sqrt{(t-6)^2} \\ &= t-6 \\ 6 \pm \sqrt{\frac{147}{4}} &= t \end{aligned}$$

$\rightarrow 12.1\text{s}$   
 $\rightarrow -0.1\text{s}$   
reject time is never negative.

- g) Compare the answers from b) - f) with those on page 323.



A rancher has 300 m of fencing with which to form a rectangular corral (an enclosure for confining livestock), one of whose sides is an existing wall which does not require fencing.

- a) If two of the sides of the rectangle are each  $x$  metres in length, show that the area of the corral can be expressed in the form  $A(x) = 300x - 2x^2$ .

- b) Use the method of completing the square to determine the maximum area possible.

- c) State the dimensions of the rectangle which gives the maximum area.



Ashley was asked by her Math teacher to find two numbers which differ by 8 and whose product is a minimum.

- a) If  $x$  represents the smaller number, write a quadratic expression in  $x$  for the product of the two numbers.

*let  $x = \text{smaller \#}$   
 $x + 8 = \text{larger \#}$*

$$P(x) = x(x+8) = x^2 + 8x$$

- b) Write the product in completed square form.

$$P(x) = (x^2 + 8x + 16) + 0 - 16$$

$$P(x) = (x+4)^2 - 16$$

$$\frac{1}{2}(8) = \boxed{4}^2 = 16$$

- c) Determine the numbers and the minimum product.

$$P = \boxed{-4}$$

$$-4 + 8 = \boxed{4}$$

*min product.*

Complete Assignment Questions #1 - #9

#1, 3, 5, 7-9