Factoring and Applications Lesson #4:
Factoring Trinomials of the Form $a(f(x))^2 + b(f(x)) + c$

In the previous lessons we have factored

- trinomials of the form $x^2 + bx + c$ by inspection
- trinomials of the form $ax^2 + bx + c$, by decomposition

In this lesson, we extend this process to consider expressions in which the variable x is replaced by a function of x.

Factoring Trinomials of the form $(f(x))^2 + b(f(x)) + c$ where $f(x)$ is a Monomial

The method of inspection can be extended to factor polynomial expressions of the form $(f(x))^2 + b(f(x)) + c$, where $f(x)$ itself is a polynomial.
In this section, we will restrict f to be a monomial.

In the trinomial $x^2 + bx + c$, the degrees of the terms are 2, 1, and 0 respectively. The method of inspection can also be used when the terms have degrees 4, 2, and 0 or 6, 3, and 0 etc.

In all cases, we make a substitution which results in a trinomial with terms of degree 2, 1, and 0.

The following example to factor $x^4 + 5x^2 + 6$ illustrates the process.

$x^4 + 5x^2 + 6$ can be written $(x^2)^2 + 5(x^2) + 6$.

Make the substitution $A = x^2$ so the expression becomes $A^2 + 5A + 6$, which factors to $(A + 2)(A + 3)$.

Replace A by x^2 to get $(x^2 + 2)(x^2 + 3)$. Then $x^4 + 5x^2 + 6 = (x^2 + 2)(x^2 + 3)$.

With experience, this process can be done by inspection.
Note that in this example the function $f(x) = x^2$.

Class Ex. #1

Factor completely.

a) $a^4 - 5a^2 - 14$

let $n = a^2$

$n^2 - 5n - 14$

replace!!

$(n-7)(n+2)$

$\boxed{(a^2-7)(a^2+2)}$

b) $x^4 + 4x^2 - 5$

let $n = x^2$

$n^2 + 4n - 5$

$(n+5)(n-1)$

$(x^2+5)(x^2-1)$

$\boxed{(x^2+5)(x+1)(x-1)}$

c) $x^6 - 9x^3 + 14$

Factoring Trinomials of the form $a(f(x))^2 + b(f(x)) + c$ *where f(x) is a Monomial*

The method of decomposition can be extended to factor polynomial expressions of the form $af(x)^2 + b(f(x)) + c$ where f itself is a polynomial.
In this section, we will restrict f to be a monomial.

In the trinomial $ax^2 + bx + c$, the degrees of the terms are $2, 1,$ and 0 respectively.
The method of decomposition can also be used when the terms have degrees $4, 2,$ and 0 or $6, 3,$ and 0 etc.

The expression $4y^4 - 11y^2 - 3$ can be factored using the method of decomposition by substituting $A = y^2$ or by splitting $-11y^2$ into two terms in y^2.

Complete the work started below.

Method 1

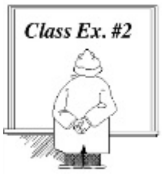
$4y^4 - 11y^2 - 3 = 4(y^2)^2 - 11(y^2) - 3$

Let $A = y^2$ $\quad 4A^2 - 11A - 3$

$=$

Method 2

$4y^4 - 11y^2 - 3 = 4y^4 - 12y^2 + 1y^2 - 3$

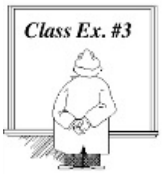


Class Ex. #2

Factor completely.

a) $4x^4 - 5x^2 - 6$

b) $2a^2b^2 - 31ab + 99$



Class Ex. #3

Factor completely the expression $8x^4 + 10x^2 - 3$.

let $n = x^2$

$8n^2 + 10n - 3$

-24

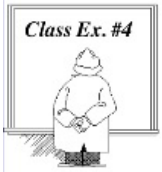
$8n^2 - 2n + 12n - 3$

$2n(4n-1) + 3(4n-1)$

$(4n-1)(2n+3)$

$(4x^2-1)(2x^2+3)$

$\boxed{(2x-1)(2x+1)(2x^2+3)}$



Given that $(\sin x)^2$ is written as $\sin^2 x$ and $(\cos x)^2$ is written as $\cos^2 x$, factor

a) $6\sin^2 x - 7\sin x + 2$ **b)** $4\cos^2 x + 11\cos x - 3$

let $n = \sin x$

$$\underbrace{6n^2 - 7n + 2}_{12}$$

$$6n^2 - 4n - 3n + 2$$

$$2n(3n-2) - 1(3n-2)$$

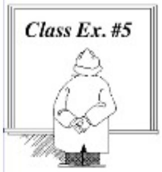
$$(3n-2)(2n-1)$$

Complete Assignment Questions #1 - #4

$$\boxed{(3\sin x - 2)(2\sin x - 1)}$$

Factoring Trinomials of the form $a(f(x))^2 + b(f(x)) + c$ ***where f(x) is a Binomial***

do not expand!



Factor.

a) $7(x-3)^2 - 4(x-3) - 3$ **b)** $9(a+4)^2 + (a+4) - 10$

let $n = x - 3$

$$\underbrace{7n^2 - 4n - 3}_{-21}$$

$$7n^2 - 7n + 3n - 3$$

$$7n(n-1) + 3(n-1)$$

$$(7n+3)(n-1)$$

$$(7(x-3)+3)((x-3)-1)$$

$$\boxed{(7x-18)(x-4)}$$

let $A = x+3$
let $B = x+5$

$$4(x+3)^2 - 9(x+5)^2$$

$$4A^2 - 9B^2$$

$$(2A-3B)(2A+3B)$$

Complete Assignment Questions #5 - #10

$\boxed{\#1\text{-}5\ (a, c, e\ldots),\ 6,\ 9}$