Operations on Radicals Lesson #3:
Dividing Radicals - Part One

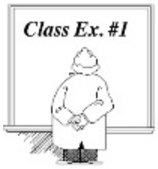
Dividing Radicals

In previous work, we discovered that $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$, $a \geq 0$, $b > 0$, and $a, b \in R$.

We can use this rule to divide radicals of the form $\dfrac{m\sqrt{a}}{n\sqrt{b}}$.

To divide radicals, the index must be the same in each radical.

- Divide numerical coefficients by numerical coefficients.
- Divide radicand by radicand.
- Simplify into mixed radical form if possible.



Divide and simplify where possible.

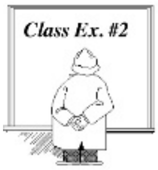
a) $\dfrac{\sqrt{30}}{\sqrt{6}} = \boxed{\sqrt{5}}$

b) $\dfrac{8\sqrt[3]{21}}{2\sqrt[3]{3}} = \boxed{4\sqrt[3]{7}}$

c) $\dfrac{\overset{3}{\cancel{15}}\sqrt{48}}{\underset{2}{\cancel{10}}\sqrt{6}} = \dfrac{3}{2}\sqrt{8} = \dfrac{3}{2}\cdot 2\sqrt{2} = \boxed{3\sqrt{2}}$

d) $\dfrac{\overset{1}{\cancel{4}}\sqrt{ab}}{\underset{3}{\cancel{12}}\sqrt{a}} = \boxed{\dfrac{1}{3}\sqrt{b}}$

In some cases, converting a radical into its simplest mixed radical form before dividing will make the calculation easier.



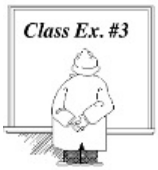
Simplify numerator and denominator, then divide.

a) $\dfrac{4\sqrt{54}}{3\sqrt{8}}$

b) $\dfrac{8\sqrt{126}}{\sqrt{112}}$

c) $\dfrac{\overset{1}{\cancel{10}}\sqrt[3]{162}}{\underset{2}{\cancel{20}}\sqrt[3]{128}}$ $162 \rightarrow \sqrt[3]{27},\ \sqrt[3]{6}$ $128 \rightarrow \sqrt[3]{64},\ \sqrt[3]{2}$

$\dfrac{3\sqrt[3]{6}}{8\sqrt[3]{2}} = \boxed{\dfrac{3}{8}\sqrt[3]{3}}$



Divide each term in the numerator by the denominator, and simplify.

$$\frac{\sqrt{24}+\sqrt{48}-\sqrt{108}}{\sqrt{6}}$$

$= \sqrt{4} + \sqrt{8} - \sqrt{18}$

$= 2 + 2\sqrt{2} - 3\sqrt{2}$

$= \boxed{2-\sqrt{2}}$

Complete Assignment Questions #1 - #4

Rationalizing the Denominator

Usually answers are written in **simplest form**, e.g. $\frac{1}{6}+\frac{1}{3}=\frac{3}{6}$ which simplifies to $\frac{1}{2}$.

In the division of radicals in this unit, regard simplest form as the form in which

i) the denominator of the fraction is a rational number, i.e. it does not contain a radical

ii) the radicand cannot contain a fraction and is expressed in simplest mixed form

The process of eliminating the radical from the denominator (i.e. converting the denominator from an irrational number to a rational number) is called **rationalizing the denominator**. The denominators in this lesson are all of monomial form. Denominators in binomial form will be discussed in the next lesson.

Class Ex. #4

Simplify by rationalizing the denominator.

a) $\frac{1}{\sqrt{13}}$ b) $\frac{\sqrt{5}}{\sqrt{2}}$ c) $\frac{\sqrt{2}}{-\sqrt{6}}$ d) $\frac{\sqrt{20}}{\sqrt{3}}$

$$=\frac{2\sqrt{5}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\boxed{\frac{2\sqrt{15}}{3}}$$

Class Ex. #5

Simplify.

a) $\frac{7}{3\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}$ b) $\sqrt{\frac{18}{5}}$ c) $\frac{3\sqrt{12}}{\sqrt{72}}$

a) $$=\frac{\cancel{7}^{1}\sqrt{7}}{\cancel{21}_{3}}=\boxed{\frac{\sqrt{7}}{3}}$$

b) $$=\frac{\sqrt{18}}{\sqrt{5}}=\frac{3\sqrt{2}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\boxed{\frac{3\sqrt{10}}{5}}$$

Class Ex. #6

Simplify the radical expression $\frac{3\sqrt{18}-\sqrt{12}}{\sqrt{2}}$ by

a) rationalizing the denominator b) dividing numerator and denominator by $\sqrt{2}$

a) $$\frac{3\sqrt{18}-\sqrt{12}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{3\sqrt{36}-\sqrt{24}}{2}$$

$$=\frac{^{9}\cancel{18}-{}^{1}\cancel{2}\sqrt{6}}{\cancel{2}_{1}}$$

$$=\boxed{9-\sqrt{6}}$$

b) $$\frac{3\sqrt{\cancel{18}_{9}}-\sqrt{\cancel{12}_{6}}}{\sqrt{\cancel{2}}}=\boxed{9-\sqrt{6}}$$

Complete Assignment Questions #5 - #16

$\boxed{\#1-7\ (a,c)}$