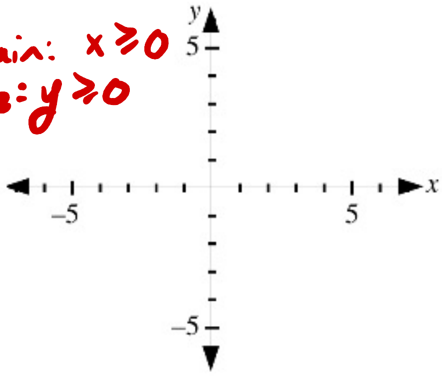
Factoring and Applications Lesson #7:
Solving Radical Equations Using Factoring

Restrictions on Values for the Variable in a Radical Expression

a) Consider the radical expression $\sqrt{x}$.

Domain: $x \geq 0$
Range: $y \geq 0$



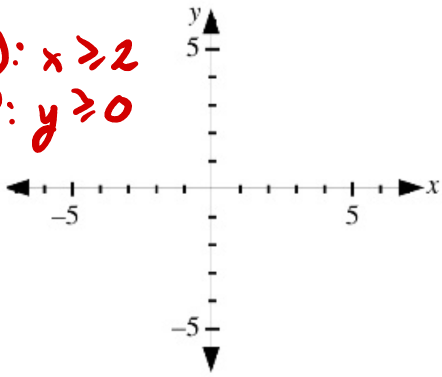
i) On the grid, sketch the graph of $y = \sqrt{x}$.

ii) State the domain of the graph of $y = \sqrt{x}$.

iii) For what values of x is the radical expression $\sqrt{x}$ defined?

b) Consider the radical expression $\sqrt{x-2}$.

D: $x \geq 2$
R: $y \geq 0$



i) On the grid, sketch the graph of $y = \sqrt{x-2}$.

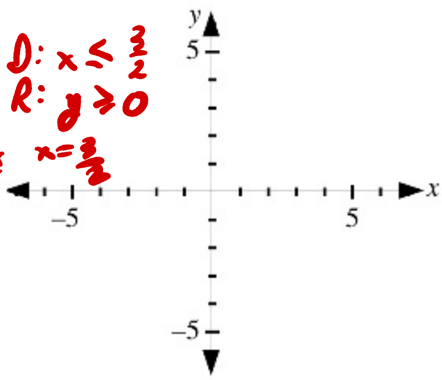
ii) State the domain of the graph of $y = \sqrt{x-2}$.

iii) For the radical expression $\sqrt{x-2}$, state the restrictions on x.

iv) Explain how to determine the restrictions algebraically.

c) Consider the radical expression $\sqrt{3-2x}$.

D: $x \leq \frac{3}{2}$
R: $y \geq 0$



i) On the grid, sketch the graph of $y = \sqrt{3-2x}$.

$3-2x=0$
$\frac{3}{2} = \frac{2x}{2}$
$x = \frac{3}{2}$

ii) State the restrictions of the variable in the radical expression $\sqrt{3-2x}$.

iii) Determine the restrictions algebraically.

Radical Equations

A radical equation is an equation which contains a radical. A value of the variable which satisfies the equation is called a **root** of the equation.

In this lesson we
- will solve radical equations graphically and algebraically
- focus on equations involving one radical only and provide as enrichment equations involving two radicals
- use factoring quadratic equations as one of the aglebraic techniques to solve radical equations

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Class Ex. #1

Algebraically, determine any restrictions on values for the variable in these radical equations.

a) $\sqrt{4x+1} = 8$ $\boxed{x \geq -\frac{1}{4}}$

$4x+1=0$

$\frac{4x}{4} = \frac{-1}{4}$

$x = -\frac{1}{4}$

b) $\sqrt{2-5x} - 7 = 0$ $\boxed{x \leq \frac{2}{5}}$

$2-5x=0$

$\frac{2}{5} = \frac{5x}{5}$

$x = \frac{2}{5}$

c) $12 - \sqrt{24-5x} = 3$

$24-5x=0$

$\frac{24}{5} = \frac{5x}{5}$

$\boxed{x \leq \frac{24}{5}}$

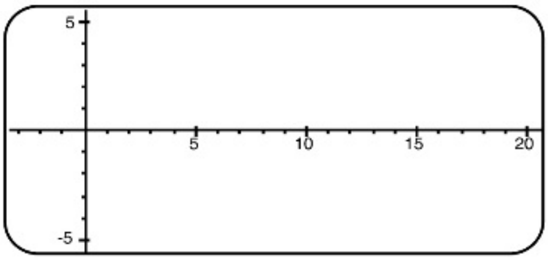
Solving Radical Equations Graphically

Class Ex. #2

Consider the radical equation $\sqrt{x+1} = 4$.

a) State the values of x for which the radical equation is defined.

b) Explain how to use a graphing calculator to find the solution to the equation by using the intersect feature of the calculator.



c) Use the method in b) to solve the radical equation $\sqrt{x+1} = 4$. Use a window $x:[-3, 20, 1]$ $y:[-5, 5, 1]$ and label the displayed graphs on the grid above.

d) State the solution to the equation.

e) Verify the solution algebraically.

Complete Assignment Questions #1 - #4

In Investigation 2, one of the solutions can be verified but the other solution cannot be verified. Can you explain why solving a radical equation by squaring both sides of the equation can lead to an invalid solution? The answer is provided in the section below.

Extraneous Roots

The process used in solving Investigation 2 led to an answer which is not one of the roots of the original radical equation, or of the radical equation in step 1. It is, however, a root of the non-radical equation in step 2 formed by squaring the original equation.

This is because the process of solving a radical equation is based on squaring both sides of the equation. If two quantities are equal, then their squares are equal. The converse, however, is not necessarily true. Two quantities which have squares that are equal are not necessarily equal quantities, e.g. $(-3)^2 = (3)^2$ but $-3 \neq 3$,

In general, the process of squaring may lead to an answer which does not satisfy the original equation. This type of an answer is called an **extraneous solution** or **extraneous root**.

For this reason, it is always necessary to verify the solution to a radical equation solved algebraically.

Solving Radical Equations Algebraically

Use the following method to solve radical equations algebraically.

Step 1: Isolate the radical term.
(Enrichment: If there are two radical terms, isolate the more complex term)

Step 2: Square both sides of the equation.

Step 3: Solve the resulting equation.
(Enrichment: If the resulting equation contains a radical term, repeat steps 1 & 2)

Step 4: Verify all answers because the squaring in step 2 may result in **extraneous roots.**

Class Ex. #3

Solve the following radical equations.

a) $\sqrt{a+10} - 4 = a$

$(\sqrt{a+10})^2 = (a+4)^2$

$a+10 = a^2+8a+16$

$0 = a^2+7a+6$

$0 = (a+1)(a+6)$

$\boxed{a = -1, \cancel{6}}$ ext

b) $(\sqrt{-1-2x})^2 = (x)^2$

$-1-2x = x^2$

$0 = x^2+2x+1$

$0 = (x+1)(x+1)$

$\boxed{x = \cancel{-1}}$ ext.

no solution $\emptyset$

Complete Assignment Question #5 - #11

$\boxed{\#1\text{–}11}$

$$\sqrt{x+1} - \sqrt{2x+3} = 0$$

$$\left(\sqrt{x+1}\right)^2 = \left(\sqrt{2x+3}\right)^2$$

$$x+1 = 2x+3$$

$$\cancel{-2} = x$$

ext.

no Solution.