

# Quadratic Functions and Equations Lesson #1: Connecting Zeros, Roots, and x-intercepts

## Function and Function Notation

Recall the following information from previous math courses:

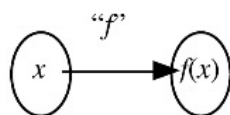
### Function

A functional relation, **or function**, is a special type of relation in which each element of the domain is related to exactly one element of the range. If any element of the domain is related to more than one element of the range, then the relation is not a function.

one "x" value in  
→ one "y" value out.

### Function Notation

Under a function,  $f$ , the image of an element,  $x$ , in the domain is denoted by  $f(x)$ , which is read "f of x".



Consider a function  $f$  defined by the formula  $f(x) = 4x - 8$ .

The notation  $f(x) = 4x - 8$  is called **function notation**.

We can show that, under the function  $f$ , the image of 5 is 12. We write  $f(5) = 12$ .

#### function notation

$$\begin{aligned} f(x) &= 4x - 8 \\ f(5) &= 4(5) - 8 \\ f(5) &= 12 \end{aligned}$$

#### equation of graph of function

$$\begin{aligned} y &= 4x - 8 \\ y &= 4(5) - 8 \\ y &= 12 \end{aligned}$$

We can also show that, under the function  $f$ , the image of 2 is 0. We write  $f(2) = 0$ .

#### function notation

$$\begin{aligned} f(x) &= 4x - 8 \\ f(2) &= 4(2) - 8 \\ f(2) &= 0 \end{aligned}$$

#### equation of graph of function

$$\begin{aligned} y &= 4x - 8 \\ y &= 4(2) - 8 \\ y &= 0 \end{aligned}$$

We can say:

"The zero of the function  $f(x) = 4x - 8$  is 2." "The root of the equation  $y = 4x - 8$  is 2."

## Zero(s) of a Function

A **zero of a function** is a value of the independent variable which makes the value of the function equal to zero. Zero(s) of a function can be found by solving the equation  $f(x) = 0$ .



Find the zero of the function  $f$  where  $f(x) = 7x - 21$ .

let  $f(x) = 0$  \*  $f(x) = y$

$$\begin{aligned} 0 &= 7x - 21 \\ 0 &= 7(x - 3) \end{aligned}$$

$x = 3$

root of the equation  
zero of the function  
x-intercept of the graph.

**Investigation #2**
*Connecting Roots, x-intercepts, and Zeros in a Quadratic Relation*

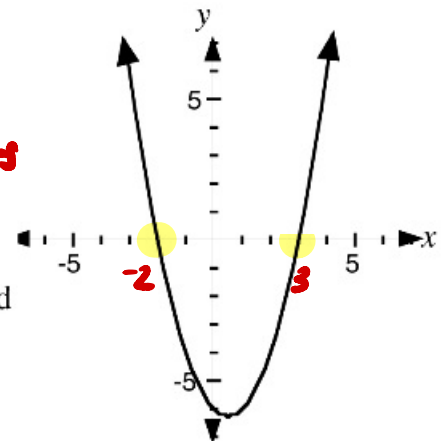
- a) Jacques is determining the roots of the equation  $x^2 - x - 6 = 0$ . He wrote the equation in factored form and used the Zero Product Law to determine the roots of the equation.

Complete his work to solve for  $x$ .

$$\begin{aligned} x^2 - x - 6 &= 0 \\ (x - 3)(x + 2) &= 0 \\ x &= 3, -2 \quad \text{Roots.} \end{aligned}$$

- b) The graph of  $y = x^2 - x - 6$  is shown. State the  $x$ -intercepts of the graph and mark them on the grid.

$$x = 3, -2 \quad \text{x-intercepts}$$



- c) i) State the connection between the **x-intercepts** of the graph of  $y = x^2 - x - 6$  and the **roots** of the equation  $x^2 - x - 6 = 0$ .

Same values.

- ii) Explain the connection between the **factors** of  $x^2 - x - 6$  and the **roots** of the equation  $x^2 - x - 6 = 0$ .

factors  $\rightarrow$  roots  
= 0

- d) Consider the function  $g(x) = x^2 - x - 6$ . Determine the **zeros** of the function.

$$\begin{aligned} 0 &= x^2 - x - 6 \\ 0 &= (x - 3)(x + 2) \\ x &= 3, -2 \quad \text{zeros} \end{aligned}$$

- e) State the connection between
- the **x-intercepts** of the graph of  $y = x^2 - x - 6$
  - the **roots** of the equation  $x^2 - x - 6 = 0$ , and
  - the **zeros** of the function  $g(x) = x^2 - x - 6$

all at same values.



**Finding Zeros of a Function**

To find the zeros of a function,  $f(x)$ , either

- substitute zero for  $f(x)$  and find the roots of the resulting equation
- or
- graph the function and determine the  $x$ -intercepts of the graph

**Finding the Roots of an Equation by Factoring**

Finding the roots of a single variable equation may involve factoring. Except in the case of a linear equation, set the equation to zero before factoring.

Recall the following techniques for factoring common factors, difference of two squares, trinomials of the form  $x^2 + bx + c = 0$ , and trinomials of the form  $ax^2 + bx + c = 0$ .

Class Ex. #4



Find the roots of the following equations.

*Set  $y$ , or  $f(x) = 0$   
factor*

a)  $x^2 + 8x = 33$

$x^2 + 8x - 33 = 0$   
 $(x + 11)(x - 3) = 0$

$x = -11, 3$

b)  $6(4x + 5)(x - 3) = 0$

$x = -\frac{5}{4}, 3$

c)  $2x^2 - 8 = 0$

$2(x^2 - 4) = 0$   
 $2(x + 2)(x - 2) = 0$

$x = \pm 2$

Class Ex. #5



For the following functions

- i) find the zeros      ii) find the  $y$ -intercept of the graph of the function

a)  $f(x) = 5x^2 + 15x - 20$

$0 = 5(x^2 + 3x - 4)$   
 $0 = 5(x + 4)(x - 1)$

$x = -4, 1$

$y$ -int = -20

b)  $f(x) = 3x^2 - 11x + 10$

$y$ -int = 10  
 $0 = 3x^2 - 11x + 10$   
 $0 = (3x - 5)(x - 2)$

$x = \frac{5}{3}, 2$

c)  $g(x) = 2x(2x + 1)$

$y$ -int = 0  
 $0 = 2x(2x + 1)$

$x = 0, -\frac{1}{2}$

#1-4, 6, 8-11