Quadratic Functions and Equations Lesson #2:
Analyzing Quadratic Functions - Part One

Quadratic Function

A **quadratic function** is a function which can be written in the form

$$f(x) = ax^2 + bx + c, \text{ where } a, b, c \in R, \text{ and } a \neq 0$$

or in equation form as

$$y = ax^2 + bx + c, \text{ where } a, b, c \in R, \text{ and } a \neq 0$$

Quadratic Equation

A **quadratic equation** is an equation which can be written in the form

$$ax^2 + bx + c = 0, \text{ where } a, b, c \in R, \text{ and } a \neq 0.$$

The roots of the quadratic equation $ax^2 + bx + c = 0$ are
the zeros of the related quadratic function $f(x) = ax^2 + bx + c$.

General and Standard Forms

A quadratic function can be written in **general** or **standard** form.

General Form: $f(x) = ax^2 + bx + c$, **or** $y = ax^2 + bx + c$, where $a, b, c \in R$, and $a \neq 0$.

Standard Form: $f(x) = a(x - p)^2 + q$, **or** $y = a(x - p)^2 + q$, where $a, p, q \in R$, and $a \neq 0$.

vertex form

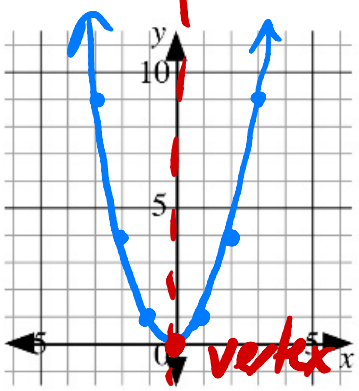
In this unit we will study both the general form and standard form, beginning with the standard form in this lesson.

Analyzing the Graph of the Function with Equation $y = x^2$

parent function

- Graph the function with equation $y = x^2$ by completing the table of values. Join the points with a smooth curve. The graph of this function is called a parabola.

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9



- The **axis of symmetry** is the "mirror" line which splits the parabola in half. State the equation of the axis of symmetry for this parabola.

  $x = 0$

- The **vertex** of a parabola is where the axis of symmetry intersects the parabola. The vertex can represent a minimum point or maximum point depending on whether the parabola opens up or down.

  Label the vertex (V) on the graph and state its coordinates.

  (0,0)

- The maximum or minimum **value** of a quadratic function occurs at the vertex and is represented by the y-coordinate of the vertex. Complete the following:

  The minimum value of the function with equation $y = x^2$ is 0.

- State the domain and range of the function with equation $y = x^2$, $x \in R$.

  Domain: $x = \mathbb{R}$ or x = all real numbers Range: $y \geq 0$

The following investigations can be completed as a class lesson or as an individual assignment. The process used in these explorations will be further developed in grade 12 mathematics.

Analyzing the Function with Equation $y = a(x - p)^2 + q$, $a = 1$

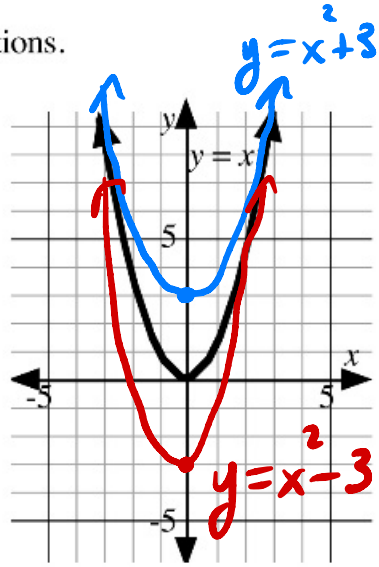
The next three investigations help us explore some general **transformations** on the graph of $y = x^2$ and the relationship they have to the standard form $y = a(x - p)^2 + q$, where $a = 1$.

A **transformation** is an operation which moves (or maps) a figure from an original position to a new position.

In each investigation, use a graphing calculator to sketch the equations.

Investigation #1 Analyzing the Graph of $y = x^2 + q$

The graph of $y = f(x) = x^2$ is shown.



a) Write an equation which represents each of the following:

- $y = f(x) + 3$ $y = x^2 + 3$
- $y = f(x) - 3$ $y = x^2 - 3$

b) Use a graphing calculator to sketch $y = f(x) + 3$ and $y = f(x) - 3$ on the grid.

c) Complete the following chart.

Function	*Equation Representing Function*	*Vertex*	*Max/Min Value*	*Equation of Axis of Symmetry*	*Description of Transformation*
$y = f(x)$	$y = x^2$	(0, 0)	min, 0	$x = 0$	no transformation
$y = f(x) + 3$	$y = x^2 + 3$	(0, 3)	min, 3	$x = 0$	vertical translation 3 units up
$y = f(x) - 3$	$y = x^2 - 3$	(0, -3)	min, -3	$x = 0$	vertical translation 3 units down
$y = f(x) + q$	$y = x^2 + q$	(0, q)	min, q	$x = 0$	vert. trans. q units up or down

d) What is the effect of the **parameter**, q, on the graph of $y = x^2 + q$?

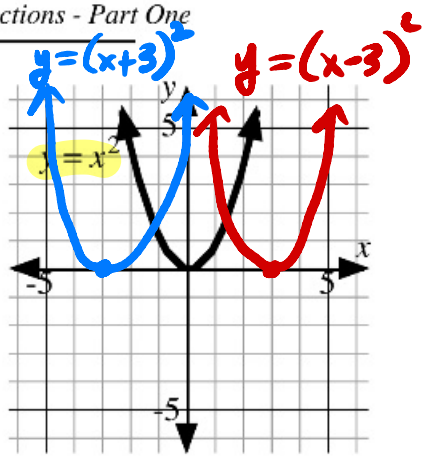
causes vertical translations up or down

e) Compared to the graph of $y = x^2$, the graph of $y = x^2 + q$ results in

a vertical translation (or shift) of q units.

If $q > 0$, the parabola moves up. If $q < 0$, the parabola moves down.

Investigation #2 Analyzing the Graph of $y = (x - p)^2$



The graph of $y = f(x) = x^2$ is shown.

a) Write an equation which represents each of the following:

- $y = f(x + 3)$ $y=(x+3)^2$ $(x-(-3))^2$ $p=-3$ left
- $y = f(x - 3)$ $y=(x-3)^2$ $p=3$ right

b) Use a graphing calculator to sketch $y = f(x + 3)$ and $y = f(x - 3)$ on the grid.

c) Complete the following chart.

Function	*Equation Representing Function*	*Vertex*	*Max/Min Value*	*Equation of Axis of Symmetry*	*Description of Transformation*
$y = f(x)$	$y = x^2$	(0, 0)	min, 0	$x = 0$	no transformation
$y = f(x + 3)$	$y=(x+3)^2$	(-3, 0)	min, 0	$x=-3$	horizontal translation 3 units left
$y = f(x - 3)$	$y=(x-3)^2$	(3, 0)	min, 0	$x=3$	horizontal translation 3 units right
$y = f(x - p)$	$y=(x-p)^2$	(p, 0)	min, 0	$x=p$	hor. trans. p units left or right

d) What is the effect of the **parameter**, p, on the graph of $y = (x - p)^2$?

causes horizontal translations left or right

e) Compared to the graph of $y = x^2$, the graph of $y = (x - p)^2$ results in a horizontal translation (shift) of p units.

If $p > 0$, the parabola moves right. If $p < 0$, the parabola moves left.

Investigation #3 Analyzing the Graph of $y = (x - p)^2 + q$

Consider the function $f(x) = x^2$.

a) Write an equation which represents $f(x + 2) - 4$. $y = (x+2)^2 - 4$ ($x - (-2)$)

b) Predict the transformations on $y = x^2$ in a). Use a graphing calculator to verify the results.

hor. translation 2 units left
vert. translation 4 units down

c) Complete the following chart.

Function	*Equation Representing Function*	*Vertex*	*Max/Min Value*	*Equation of Axis of Symmetry*	*Description of Transformation*
$y = f(x)$	$y = x^2$	(0, 0)	min, 0	$x = 0$	no transformation
$y = f(x + 2) - 4$	$y = (x+2)^2 - 4$	(-2, -4)	min, -4	$x = -2$	
$y = f(x - p) + q$	$y = (x-p)^2 + q$	(p, q)	min, q	$x = p$	hor. translation p units left or right; vert. trans. q units up or down

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Class Ex. #1

Describe how the graphs of the following functions relate to the graph of $y = x^2$.

a) $y = (x + 10)^2$ — hor. trans. 10 units left — vertex: (-10, 0)

b) $y = x^2 + 4$ — vert. trans 4 units up. — (0, 4)

c) $y + 8 = (x - 5)^2$ → $y = (x-5)^2 - 8$ — hor. trans. 5 units right. vert. trans. 8 units down — (5, -8)

Class Ex. #2

The following transformations are applied to the graph of $y = x^2$. Write the equation of the image function for each.

a) a horizontal translation of 5 units right $y = (x-5)^2$

b) a translation of 6 units down and 4 units left $y = (x+4)^2 - 6$

Class Ex. #3

Write the coordinates of the image of the point (3, 9) on the graph $y = x^2$ when a translation of two units up and seven units right is applied.

vert. trans. 2 units ↑ hor. trans. 7 units →

$(x, y) \to (?, ?) \to (x+7, y+2)$

$(3, 9) \longrightarrow (10, 11)$ * mapping *

Complete Assignment Questions #1 - #10

$$y = (x+2)^2 - 3$$

Vertex: $(-2, -3)$

eq. for axis of symmetry: $x = -2$

dir. of opening: up

Domain: $x = \mathbb{R}$

Range: $y \geq -3$

Max/Min value: min, -3

y-int: 1

x-int's (if any): $x = -2 \pm \sqrt{3}$

changes from $y = x^2$:

① hor. trans. 2 units left

② vert. trans. 3 units down

sketch:

$$0 = (x+2)^2 - 3$$
$$\pm\sqrt{3} = \sqrt{(x+2)^2}$$
$$\pm\sqrt{3} = x+2$$
$$-2 \pm \sqrt{3} = x \quad \text{(exact)}$$