Factoring and Applications Lesson #2:
Common Factors and Grouping

Binomial Common Factors

In certain circumstances, the greatest common factor may be a binomial rather than a monomial. This particular type of factoring is part of a process for factoring trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, and is covered in the next lesson.

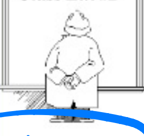
Class Ex. #1

Factor the following polynomials by removing the greatest common factor.

a) $(4x)(x+7) - (3)(x+7)$ **b)** $7(3-2y) + 2y(3-2y)$ **c)** $9a(4a+1) + (4a+1)$

$(x+7)(4x-3)$ $(3-2y)(7+2y)$ $(4a+1)(9a+1)$

Class Ex. #2



Factor the following and write the answer in simplest factored form.

a) $(3y+2)(5y+1) + (3y+2)(4y)$

$(3y+2)(5y+1+4y)$

$\boxed{(3y+2)(9y+1)}$

b) $3a(a-6) - 9(a-6)$

$(a-6)(3a-9) = \boxed{3(a-3)(a-6)}$

c) $2x(x-5) + 5(5-x)$

neg.

$2x(x-5) - 5(x-5)$

$\boxed{(x-5)(2x-5)}$

d) $20x(x-3) - 4(3-x)$

neg.

$20x(x-3) + 4(x-3)$

$(x-3)(20x+4) = \boxed{4(5x+1)(x-3)}$

Complete Assignment Question #1

Factoring by Grouping

Sometimes polynomials in four terms can be factored by removing the greatest common factor from a pair of terms followed by a binomial common factor. This method is called factoring by grouping. The method of grouping is a component of the method used to factor trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, and is covered in the next lesson.

Class Ex. #3

Factor the following polynomials by grouping.

a) $x^2 + 3x + 6x + 18$

$x(x+3) + 6(x+3)$

$\boxed{(x+3)(x+6)}$

b) $8x^2 - 2x + 12x - 3$

$2x(4x-1) + 3(4x-1)$

$\boxed{(4x-1)(2x+3)}$

c) $8a^2 - 4a - 10a + 5$

$4a(2a-1) - 5(2a-1)$

$\boxed{(2a-1)(4a-5)}$

d) $6a^2 - 9a - 2a + 3$

$3a(2a-3) - 1(2a-3)$

$\boxed{(2a-3)(3a-1)}$

e) $pq + pr - sq - sr$

$p(q+r) - s(q+r)$

$\boxed{(q+r)(p-s)}$

f) $5x^2 + 18y^2 - 15xy^2 - 6x$

$5x^2 - 15xy^2 - 6x + 18y^2$

$5x(x-3y^2) - 6(x-3y^2)$

$\boxed{(x-3y^2)(5x-6)}$

Complete Assignment Questions #2 - #4

Monomial Common Factors involving Fractions

In polynomials involving fractional coefficients, it is useful to include a fraction as part of the monomial common factor so that the remaining factor is an integral polynomial with no common factor.

eg. $\frac{1}{2}x^2 - 3x = \frac{1}{2}x(x-6)$

Such a technique will prove useful in future math courses.

Class Ex. #4

In each case, a common factor has been removed so that the remaining factor is an integral binomial. Complete the factoring and check mentally by expanding the factored form.

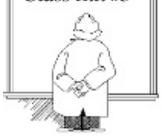
a) $\frac{1}{3}x^2 + 4x = \frac{1}{3}x(x + 12)$

b) $\frac{1}{4}a^2 - 4a = \frac{1}{4}a(a - 16)$

c) $6x + \frac{2}{3} = \frac{2}{3}(9x + 1)$

d) $\frac{1}{2}a^2 - \frac{3}{4}b^2 = \frac{1}{4}(2a^2 - 3b^2)$

Class Ex. #5



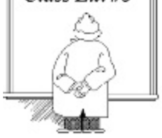
Complete the factoring and check mentally by expanding the factored form.

a) $a - \frac{1}{6}a^2 = \frac{1}{6}a(6-a)$

b) $\frac{1}{2}\pi r^2 - 2\pi r = \frac{1}{2}\pi r(r-4)$

c) $4x^2 + 2x + \frac{2}{5} = \frac{2}{5}(10x^2 + 5x + 1)$

Class Ex. #6



In each case, remove a common factor so that the remaining factor is an integral polynomial.

a) $2a + \frac{1}{4}a^3$

$\frac{1}{4}a(8 + a^2)$

b) $x^3 + \frac{1}{3}x^2 - \frac{1}{6}x$

$\frac{1}{6}x(6x^2 + 2x - 1)$

Complete Assignment Questions #5 - #14

#1-8 (a,c,e...)