Exponents and Radicals Lesson #1:
Classifying Real Numbers

Recall from previous courses the definitions of rational numbers and irrational numbers.

- Decimal numbers which repeat or terminate can be converted into fractions and are called **rational numbers**, since they can be written as the ratio of two integers.
- Decimal numbers which are both non-repeating and non-terminating cannot be converted into fractions and are called **irrational numbers**.

The set of all rational numbers and the set of all irrational numbers, when combined, form the set of **real numbers**. These numbers can be represented on a number line.

The following sets of numbers are within the real number system:

Natural Numbers
$N = \{ 1, 2, 3, \ldots \}$

Whole Numbers
$W = \{ 0, 1, 2, 3, \ldots \}$

Integers
$I = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$

Rational Numbers

$$Q = \left\{ \frac{a}{b}, \text{ where } a, b \in I,\ b \neq 0 \right\}$$

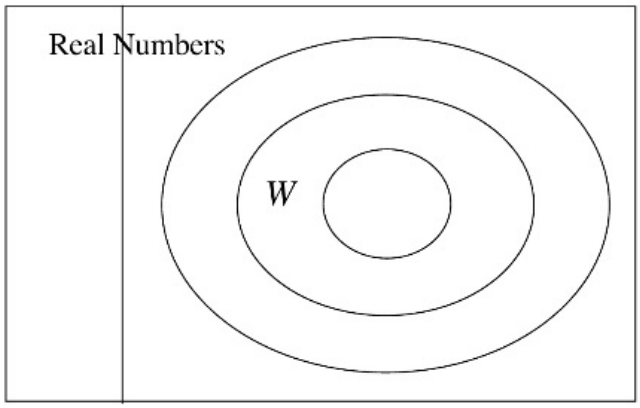
Irrational Numbers

$\overline{Q} = \{\text{non-terminating and non-repeating decimals}\}$

Real Numbers

$R = \{Q \text{ and } \overline{Q}\}$

Raven, a student in the Advanced Placement mathematics program, was asked to show the interrelationship between sets of numbers in a nested diagram. Her partial work is shown below. Complete her diagram.



Note that the area of each region bears no relation to the number of members in each set.

a) For each of the following, write all the sets of numbers to which the given number belongs. Write the answers from the largest set to the smallest set.

i) 9 real rational integers whole natural

ii) $\frac{11}{7}$ R Q

iii) $\sqrt{5}$ real irrational

iv) -7 R Q I

b) Explain why 9 belongs to five number sets, but -9 belongs to only three number sets.

-ive sign eliminates W & N.

Complete Assignment Questions #1 - #7

Square Roots

All positive numbers have two square roots: one a positive number and the other a negative number. The positive square root is called the **principal square root** and is denoted by the symbol $\sqrt{\ }$.

- The square roots of a perfect square are rational numbers.
  e.g. the square roots of 16 are 4 and -4. **NOTE:** $\sqrt{16} = 4$ only.
- The square roots of a non-perfect square are irrational numbers.
  e.g. the square roots of 17 are $\sqrt{17}$ and $-\sqrt{17}$.

perfect squares *

The ability to estimate mentally the square root of a non-perfect square is important when checking a calculator calculation for possible error.
A knowledge of some common perfect squares enables us to make such estimates to the nearest whole number.

1^2	$= 1$
2^2	$= 4$
3^2	$= 9$
4^2	$= 16$
5^2	$= 25$
6^2	$= 36$
7^2	$= 49$
8^2	$= 64$
9^2	$= 81$
10^2	$= 100$
11^2	$= 121$
12^2	$= 144$
13^2	$= 169$
14^2	$= 196$
15^2	$= 225$

In each of the following

i) estimate the value mentally (use whole numbers)

ii) use a calculator to find the decimal approximation to the nearest tenth, and decide if the estimate in i) is reasonable

a) $\sqrt{46}$ ~ 6.8

b) $-\sqrt{5}$ ~ -2.2

c) $2\sqrt{18} + 5\sqrt{37} - \sqrt{5}$

$2(4) + 5(6) - 2$

$8 + 30 - 2 = 36.5$

d) $\sqrt{\sqrt{95}}$ ~ $\sqrt{10}$ ~ 3.1

Cube Roots

All numbers (positive and negative) have one cube root, denoted by the symbol $\sqrt[3]{\ }$.

- The cube root of a perfect cube is a rational number.
  e.g. the cube root of 1 000 is 10, i.e. $\sqrt[3]{1\,000} = 10$.
  the cube root of -27 is -3, i.e. $\sqrt[3]{-27} = -3$.

- The cube root of a non-perfect cube is an irrational number.
  e.g. the cube root of 9 is $\sqrt[3]{9}$, which is irrational.

In each of the following

i) estimate the value mentally (use whole numbers)

ii) use a calculator to find the decimal approximation to the nearest tenth, and decide if the estimate in i) is reasonable

a) $\sqrt[3]{11}$ ~ 2.1

b) $\sqrt[3]{120}$ ~ 4.8

c) $4\sqrt{70} - 4\sqrt[3]{70}$

~ 4(8) − 4(4)
~ 32 − 16 ~ 16.6

perfect cubes.

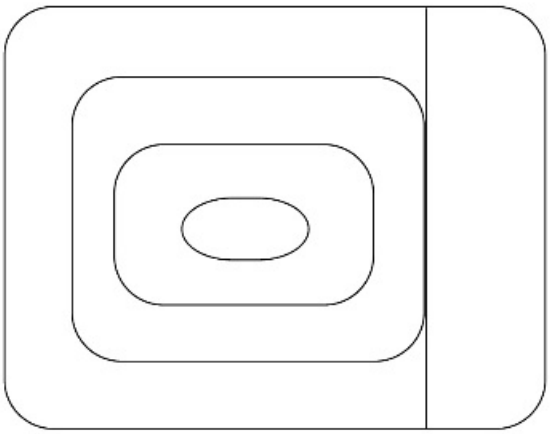
1^3	= 1
2^3	= 8
3^3	= 27
4^3	= 64
5^3	= 125
6^3	= 216
7^3	= 343
8^3	= 512
9^3	= 729
10^3	= 1000

Complete Assignment Questions #8 - #16

Assignment

2-14

1. Complete the Venn Diagram to show the interrelationship between the sets of numbers in the real number system.



2. For each of the following, write all the sets of numbers to which the given number belongs. Write the answers from the largest set to the smallest set.

a) -2 **b)** $\sqrt{36}$ **c)** 3.14159265 **d)** $-\frac{7}{5}$

e) 0 **f)** $\sqrt{7}$ **g)** $-2.1345218...$ **h)** π

3. Explain why -7 belongs to more number sets than $-\frac{7}{2}$.