Exponents and Radicals Lesson #3:
Entire Radicals and Mixed Radicals - Part One

Recall the following from Lesson #2.

i) **The product(quotient) of the roots of two numbers is equal to the root of the product (quotient) of the two numbers.**

ii) **The sum (difference) of the roots of two numbers is NOT equal to the root of the sum (difference) of the two numbers.**

In general $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ where $a, b \geq 0$ and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ where $a \geq 0, b > 0$.

In this lesson we use the above rules in reverse:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b} \text{ where } a, b \geq 0 \text{ and } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ where } a \geq 0, b > 0.$$

Write the following as a product or quotient of radicals.

a) $\sqrt{24} = \sqrt{4 \times 6} = \sqrt{4} \times \sqrt{6}$
$= 2\sqrt{6}$

b) $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2}$
$= 3\sqrt{2}$

c) $\sqrt{\dfrac{11}{4}} = \dfrac{\sqrt{11}}{\sqrt{4}} = \dfrac{\sqrt{11}}{2}$

Entire Radicals and Mixed Radicals

eg. Entire $\sqrt{24} \longleftrightarrow 2\sqrt{6}$ Mixed

Use a calculator to approximate the value of each radical to 5 decimal places.

i) $\sqrt{80}$ = 8.94427 ii) $2\sqrt{20}$ = 8.94427 iii) $4\sqrt{5}$ = 8.94427

What do you notice about the answers? Same

Complete the following to explain why the three radicals are equivalent.

$\sqrt{80} = \sqrt{4 \times 20} = \sqrt{4} \times \sqrt{20} = 2\sqrt{20}$

$\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

$\sqrt{80}$ is an example of an **entire radical;** the number is entirely under the root symbol.

$2\sqrt{20}$ and $4\sqrt{5}$ are examples of **mixed radicals**.

mixed

simplest terms *

always take out the biggest perfect square.

Entire/Pure Radicals

- Radicals expressed in the form $\sqrt[n]{b}$ are called entire (or pure) radicals.
- For example, $\sqrt{25}$, $\sqrt{80}$, $\sqrt[3]{17}$.

Mixed Radicals

- Radicals expressed in the form $a\sqrt[n]{b}$ are called mixed radicals.
- For example $\frac{2}{3}\sqrt{5}$, $8\sqrt{7}$, $-9\sqrt[3]{17}$.

Every mixed radical can be expressed as an entire radical.

To determine if an entire radical (with an index of 2) can be expressed as a mixed radical, we need to check if the number has a factor which is a perfect square.

Converting Entire Radicals (with an index of 2) to Mixed Radicals

An entire radical of index 2 may be expressed as a mixed radical when the highest perfect square has been factored out of the entire radical.

Complete the following to convert $\sqrt{108}$ to a mixed radical.

Entire Radical $\Rightarrow$ *Mixed Radical*

$$\sqrt{108} = \sqrt{\underline{36} \times 3}$$

$$= \sqrt{\underline{36}} \times \sqrt{3}$$

$$= 6 \times \sqrt{3}$$

$$\sqrt{108} = \boxed{6\sqrt{3}}$$

perfect squares.

1^2	=	1
2^2	=	4
3^2	=	9
4^2	=	16
5^2	=	25
6^2	=	36
7^2	=	49
8^2	=	64
9^2	=	81
10^2	=	100
11^2	=	121
12^2	=	144
13^2	=	169
14^2	=	196
15^2	=	225

If the perfect square which is factored out is not the highest perfect square, then the process will require more than one step to obtain the mixed radical in simplest form. When converting an entire radical to a mixed radical, it is expected that the answer will be in simplest form.

Convert the following entire radicals to mixed radicals in simplest form.

a) $\sqrt{50}$

$\sqrt{25}$ $\sqrt{2}$

$\boxed{5\sqrt{2}}$

b) $\sqrt{45}$

$\sqrt{9}$ $\sqrt{5}$

$\boxed{3\sqrt{5}}$

c) $\sqrt{320}$

$\sqrt{64}$ $\sqrt{5}$

$\boxed{8\sqrt{5}}$

Class Ex. #3

Convert the following radicals to mixed radicals in simplest form.

a) $2\sqrt{192}$

$2 \quad \sqrt{64} \quad \sqrt{3}$

$2 \cdot 8 \cdot \sqrt{3}$

$\boxed{16\sqrt{3}}$

b) $\frac{3}{4}\sqrt{160}$

$\frac{3}{4} \quad \sqrt{16} \quad \sqrt{10}$

$\frac{3}{\cancel{4}} \cdot \cancel{4} \cdot \sqrt{10}$

$\boxed{3\sqrt{10}}$

c) $\sqrt{\frac{7}{9}}$ $\frac{\sqrt{7}}{\sqrt{9}} = \boxed{\frac{\sqrt{7}}{3}}$

Complete Assignment Questions #1 - #5

Converting Entire Radicals (with an index of 3 or greater) to Mixed Radicals

An entire radical of index 3 may be expressed as a mixed radical when the highest perfect cube has been factored out of the entire radical.

perfect cubes

1^3	=	1
2^3	=	8
3^3	=	27
4^3	=	64
5^3	=	125
6^3	=	216
7^3	=	343
8^3	=	512
9^3	=	729
10^3	=	1000

Complete the following to convert $\sqrt[3]{54}$ to a mixed radical.

Entire Radical $\Rightarrow$ *Mixed Radical*

$$\sqrt[3]{54} = \sqrt[3]{27 \times 2}$$

$$= \sqrt[3]{27} \times \sqrt[3]{2}$$

$$= 3 \times \sqrt[3]{2}$$

$$\sqrt[3]{54} = \boxed{3\sqrt[3]{2}}$$

A similar process is involved for indices greater than 3.

create a list of perfects for any index

Class Ex. #4

Convert the following radicals to mixed radicals in simplest form.

a) $\sqrt[3]{6\,000}$

$\sqrt[3]{1000} \quad \sqrt[3]{6}$

$\boxed{10\sqrt[3]{6}}$

b) $\sqrt[5]{320}$

$\sqrt[5]{32} \quad \sqrt[5]{10}$

$\boxed{2\sqrt[5]{10}}$

c) $\sqrt[3]{-16}$

$\sqrt[3]{-1} \quad \sqrt[3]{8} \quad \sqrt[3]{2}$

$\boxed{-2\sqrt[3]{2}}$

$1^5 = 1$

$2^5 = 32$

$3^5 = 243$

$4^5 = 1024$

Complete Assignment Questions #6 - #9

Extension: Radicals involving Variables (Entire to Mixed)

Since $x^3 \times x^3 = x^6$, then $\sqrt{x^6} =$ ___ . Also, since $x^5 \times x^5 \times x^5 = x^{15}$ then $\sqrt[3]{x^{15}} =$ ___.

So $\sqrt{x^4} =$ ____. $\sqrt{y^{10}} =$ ____. $\sqrt{a^8b^6} =$ _____. $\sqrt[3]{x^{24}} =$ ____. $\sqrt[3]{y^6} =$ ____.

Complete the following to convert $\sqrt{x^5}$ to a mixed radical.

Entire Radical $\Rightarrow$ *Mixed Radical*

$$\sqrt{x^5} = \sqrt{x^4 \times x}$$
$$= \sqrt{\quad} \times \sqrt{x}$$
$$= \_\_\_\_ \times \sqrt{x}$$
$$\sqrt{x^5} =$$

Class Ex. #5

Convert the following entire radicals to mixed radicals in simplest form.

a) $\sqrt{a^7}$ **b)** $\sqrt{t^9}$ **c)** $\sqrt[3]{x^5}$ **d)** $\sqrt[3]{x^7}$

a) $\sqrt{a^2}\ \sqrt{a^2}\ \sqrt{a^2}\ \sqrt{a}$

$a \cdot a \cdot a \cdot \sqrt{a}$

$\boxed{a^3\sqrt{a}}$

b) $\sqrt{t^8}\ \sqrt{t}$

$\boxed{t^4\sqrt{t}}$

c) $\sqrt[3]{x^3}\ \sqrt[3]{x^2}$

$\boxed{x\sqrt[3]{x^2}}$

d) $\sqrt[3]{x^6}\ \sqrt[3]{x}$

$\boxed{x^2\sqrt[3]{x}}$

Class Ex. #6

Convert the following entire radicals to mixed radicals in simplest form.

a) $\sqrt{x^6y^5}$ **b)** $\sqrt{18x^3}$ **c)** $\sqrt{32y^7z^8}$ **d)** $\sqrt[3]{40x^4y^9}$

a) $\boxed{x^3y^2\sqrt{y}}$

b) $\boxed{3x\sqrt{2x}}$

c) $\boxed{4y^3z^4\sqrt{2y}}$

d) $\sqrt[3]{40}\ \sqrt[3]{x^4}\ \sqrt[3]{y^9}$

$\sqrt[3]{8}\ \sqrt[3]{5}\ \sqrt[3]{x^3}\ \sqrt[3]{x}\ y^3$

$2 \cdot \sqrt[3]{5} \cdot x \cdot \sqrt[3]{x} \cdot y^3$

$\boxed{2xy^3\sqrt[3]{5x}}$

Complete Assignment Questions #10 - #12

#1-10 (a,c,e...)