Exponents and Radicals Lesson #6:
Rational Exponents - Part One

Review of the Exponent Laws

The exponent laws with integral exponents and numerical and variable bases were covered in previous math courses.

Complete the table as a review of the exponent laws.

Numerical Bases	Variable Bases	Exponent Laws
$8^3 \times 8^2 = (8 \cdot 8 \cdot 8)(8 \cdot 8)$ $= 8^5$ or 8^{3+2}	$a^3 \times a^2 = (a \cdot a \cdot a)(a \cdot a)$ $= a^5$ or a^{3+2}	**Product Law** $(a^m)(a^n) = a^{m+n}$
$8^3 \div 8^2 = \frac{8 \cdot 8 \cdot 8}{8 \cdot 8}$ $= 8^1$ or 8^{3-2}	$a^3 \div a^2 = \frac{\cancel{a} \cdot \cancel{a} \cdot a}{\cancel{a} \cdot \cancel{a}}$ $= a^1$ or a^{3-2}	**Quotient Law** $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$
$(8 \cdot 7)^3 = (8 \cdot 7)(8 \cdot 7)(8 \cdot 7)$ $= (8 \cdot 8 \cdot 8)(\qquad)$ $= 8^3 \cdot 7^3$	$(a \cdot b)^3 = (a \cdot b)(a \cdot b)(a \cdot b)$ $= (a \cdot a \cdot a)(b \cdot b \cdot b)$ $= a^3 b^3$	**Power of a Product Law** $(ab)^m = a^m b^m$ $(2a)^2 = 4a^2$
$\left(\frac{8}{7}\right)^3 = \left(\frac{8}{7}\right)\left(\frac{8}{7}\right)\left(\frac{8}{7}\right)$ $= \frac{8^3}{7^3}$	$\left(\frac{a}{b}\right)^3 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)$ $= \frac{a^3}{b^3}$	**Power of a Quotient Law** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $(b \neq 0)$
$(8^3)^2 = (8^3)(8^3)$ $= (8 \cdot 8 \cdot 8)(8 \cdot 8 \cdot 8)$ $= 8^6$ or $8^{3 \times 2}$	$(a^3)^2 = (a^3)(a^3)$ $= (a \cdot a \cdot a)(a \cdot a \cdot a)$ $= a^6$ or $a^{3 \times 2}$	**Power of a Power Law** double power raise $(a^m)^n = a^{m \cdot n}$

2. a) Evaluate i) $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2 = \left(\sqrt[3]{8}\right)^2 =$ ii) $8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = \sqrt[3]{(8^2)} =$

b) Which of the calculations above is the easier method for evaluating $8^{\frac{2}{3}}$?

c) Write the following in radical form and evaluate manually. Verify with a calculator.

i) $64^{\frac{3}{2}} =$ **ii)** $4^{\frac{5}{2}}$ **iii)** $81^{\frac{3}{4}}$

3. a) Use exponent laws to simplify $8^{\frac{2}{3}} \times 8^{-\frac{2}{3}}$.

b) Use the result in a) to write $8^{-\frac{2}{3}}$ in a form with a positive exponent.

Evaluate $8^{-\frac{2}{3}}$ without using a calculator.

Rational Exponents

$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $a^{\frac{m}{n}} = \sqrt[n]{a^m}$, $m \in I,\ n \in N,\ a \neq 0$ when m is 0.

Note that if n is even, then a must be non-negative.

$a^{-\frac{m}{n}} = \dfrac{1}{\left(\sqrt[n]{a}\right)^m}$ or $a^{-\frac{m}{n}} = \dfrac{1}{\sqrt[n]{a^m}}$, $m \in I,\ n \in N,\ a \neq 0$ when m is 0.

Note that if n is even, then a must be positive.

Class Ex. #1

Write the following in radical form and evaluate without using a calculator.
Verify with a calculator.

a) $25^{\frac{3}{2}}$

$\left(\sqrt{25}\right)^3$

$= 5^3 = \boxed{125}$

b) $1000^{\frac{4}{3}}$

$\left(\sqrt[3]{1000}\right)^4$

$= 10^4$

$= \boxed{10000}$

c) $27^{-\frac{2}{3}}$

$\left(\frac{1}{27}\right)^{\frac{2}{3}}$

$\left(\sqrt[3]{\frac{1}{27}}\right)^2$

$= \left(\frac{1}{3}\right)^2 = \boxed{\frac{1}{9}}$

d) $16^{-\frac{3}{4}}$

$\left(\frac{1}{16}\right)^{\frac{3}{4}}$

$\left(\sqrt[4]{\frac{1}{16}}\right)^3$

$= \left(\frac{1}{2}\right)^3 = \boxed{\frac{1}{8}}$

e) $(-8)^{\frac{2}{3}}$

$\left(\sqrt[3]{-8}\right)^2$

$= (-2)^2 = \boxed{4}$

f) $-8^{\frac{2}{3}}$

$-1 \quad 8^{\frac{2}{3}}$

$-1 \cdot \left(\sqrt[3]{8}\right)^2$

$= -1 \cdot 4 = \boxed{-4}$

g) $(3^2 + 4^2)^{\frac{1}{2}}$

$(25)^{\frac{1}{2}}$

$= \sqrt{25}$

$= \boxed{5}$

Class Ex. #2

Write the following in radical form and evaluate without using a calculator. Verify with a calculator.

a) $\left(\frac{9}{4}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{9}{4}}\right)^3 = \left(\frac{3}{2}\right)^3 = \boxed{\frac{27}{8}}$

b) $\left(\frac{9}{4}\right)^{-\frac{3}{2}} = \left(\frac{4}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{4}{9}}\right)^3 = \left(\frac{2}{3}\right)^3 = \boxed{\frac{8}{27}}$

Complete Assignment Questions #1 - #5

Class Ex. #3

Write an equivalent expression using radicals.

a) $r^{\frac{1}{3}} = \boxed{\sqrt[3]{r}}$

b) $s^{\frac{4}{7}} = \boxed{\left(\sqrt[7]{s}\right)^4}$

c) $t^{-\frac{1}{6}} = \boxed{\sqrt[6]{\frac{1}{t}}}$ or $= \frac{\sqrt[6]{1}}{\sqrt[6]{t}} = \frac{1}{\sqrt[6]{t}}$

d) $v^{-\frac{3}{2}} = \boxed{\left(\sqrt{\frac{1}{v}}\right)^3} = \left(\frac{\sqrt{1}}{\sqrt{v}}\right)^3 = \frac{1}{\left(\sqrt{v}\right)^3}$

Class Ex. #4

Consider the following powers.

A. $64^{\frac{2}{3}}$ **B.** $(-64)^{\frac{2}{3}}$ **C.** $64^{\frac{3}{2}}$ **D.** $(-64)^{\frac{3}{2}}$

Explain why three of the above powers can be calculated but the other has no meaning.

Class Ex. #5

A cube has a volume of 60 m^3.

a) Write a power which represents the edge length of the cube.

b) Write a power which represents the surface area of the cube.

c) Use a calculator to calculate the edge length and surface area to the nearest tenth.

Class Ex. #6

Write the number 10 in the following forms:

a) as a power with an exponent of $\frac{1}{2}$ **b)** as a power with an exponent of $\frac{1}{3}$

Complete Assignment Questions #6 - #13 #1-4, 6

+ review for Wednesday's Radicals Quiz