Exponents and Radicals Lesson #7:
Rational Exponents - Part Two

Review | Complete the following as a review.

Product Law $\qquad x^m x^n =$ x^{m+n}

Quotient Law $\qquad x^m \div x^n =$ x^{m-n}

Power of a Power $\qquad (x^m)^n =$ x^{mn}

Power of a Product $\qquad (xy)^m =$ $x^m y^n$

Power of a Quotient $\qquad \left(\dfrac{x}{y}\right)^m =$ $\dfrac{x^m}{y^m}$ $, y \neq 0$

Integral Exponent Rule $\qquad x^{-m} = \left(\dfrac{1}{x}\right)^m$ $, where $x \neq 0$

Rational Exponents $\qquad x^{\frac{m}{n}} =$ $\sqrt[n]{x^m}$ $or $(\sqrt[n]{x})^m$

zero exponent law $\qquad x^0 = 1$

eg. $(2a)^0 = 1$

$2a^0 = 2$

Writing Powers as Radicals

Class Ex. #1 | Write each power as a radical.

a) $x^{\frac{1}{6}}$ $\qquad$ **b)** $-y^{\frac{5}{4}}$ $\qquad$ **c)** $(-z)^{\frac{5}{3}}$ $\qquad$ **d)** $(-z)^{-\frac{5}{3}}$ $\qquad$ **e)** $5t^{\frac{3}{4}}$ $\qquad$ **f)** $(5t)^{\frac{3}{4}}$

a) $\sqrt[6]{x}$
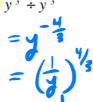
b) $-\left(\sqrt[4]{y}\right)^5$

Class Ex. #2 | Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

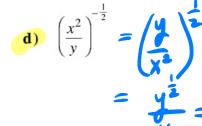**a)** $x^{\frac{3}{2}} \times x^1$ $\qquad$ **b)** $y^{\frac{1}{3}} \div y^{\frac{5}{3}}$ $\qquad$ **c)** $(a^{\frac{1}{2}})^{\frac{2}{3}}$ $\qquad$ **d)** $\left(\dfrac{x^2}{y}\right)^{-\frac{1}{2}}$

a) $= x^{5/2}$
$= \left(\sqrt{x}\right)^5$

b) $= y^{-\frac{4}{3}}$
$= \left(\dfrac{1}{y}\right)^{4/3}$
$= \left(\sqrt[3]{\dfrac{1}{y}}\right)^4$

d) $= \left(\dfrac{y}{x^2}\right)^{\frac{1}{2}}$
$= \dfrac{y^{\frac{1}{2}}}{x} = \dfrac{\sqrt{y}}{x}$

Class Ex. #3 | Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

a) $4x^{\frac{3}{4}} \times 3x^{-\frac{1}{2}}$ $\qquad$ **b)** $\dfrac{5x^{\frac{3}{5}}}{25x^{-\frac{3}{5}}}$ $\qquad$ **c)** $(8a^{\frac{1}{2}})^{\frac{4}{3}}$

a) $= 12x^{\frac{3}{4}+(-\frac{1}{2})}$
$= 12x^{\frac{1}{4}} = \boxed{12\sqrt[4]{x}}$

c) $8^{\frac{4}{3}} a^{\frac{2}{3}}$
$\left(\sqrt[3]{8}\right)^4 \left(\sqrt[3]{a}\right)^2$
$= 16\left(\sqrt[3]{a}\right)^2$

Complete Assignment Questions #1 - #3

Writing Radicals as Powers

We can use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ to write radicals as powers.

Class Ex. #4

Write each radical as a power in the form a^n, $n \in Q$.

a) $\sqrt[3]{a^5}$ b) $\sqrt[5]{a^2}$ c) $\sqrt{a^9}$ d) $\dfrac{1}{\sqrt{a^7}}$

$= a^{\frac{5}{3}}$

$= a^{9/2}$

Class Ex. #5

Write as a power and evaluate.

a) $\sqrt{\sqrt{1296}}$ b) $\dfrac{1}{\sqrt{169}}$ c) $\sqrt[3]{\sqrt{64}}$

$= \left(1296^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= 1296^{\frac{1}{4}}$

$= 6$

Class Ex. #6

Write each expression in the form ax^n, where $a \in I$, and $n \in Q$.

a) $\sqrt[3]{8x^5}$ b) $\sqrt[5]{32x^3}$ c) $\sqrt{900x}$

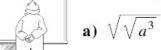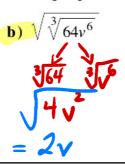$\sqrt[3]{8} \quad \sqrt[3]{x^5}$

$= 2x^{5/3}$

d) $\left(\sqrt[3]{x^5}\right)\left(\sqrt[3]{x}\right)$ e) $2\sqrt{x} \times \sqrt[3]{x}$

Class Ex. #7

Write an equivalent expression using exponents.

a) $\sqrt{\sqrt{a^3}}$

b) $\sqrt{\sqrt[3]{64v^6}}$

$\sqrt[3]{64} \quad \sqrt[3]{v^6}$

$\sqrt{4v^2}$

$= 2v$

c) $\left(\sqrt[4]{x^5 y^3}\right)^{\frac{3}{2}}$

Complete Assignment Questions #4 - #15

Assignment #1-7 (a,c,e...)

1. Write each power as an entire radical.

a) $a^{\frac{4}{5}}$

b) $b^{\frac{3}{2}}$

c) $c^{\frac{1}{4}}$

d) $x^{-\frac{2}{5}}$

e) $y^{-\frac{1}{3}}$

f) $5h^{\frac{2}{3}}$

g) $(5h)^{\frac{2}{3}}$

h) $-r^{\frac{5}{4}}$

i) $(-r)^{\frac{5}{4}}$

j) $2x^{-\frac{1}{2}}$

2. Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

a) $x^{\frac{7}{2}} \times x$

b) $y^{\frac{6}{5}} \div y^{\frac{4}{5}}$

c) $(a^{\frac{2}{5}})^{\frac{3}{4}}$

d) $(e^3 f)^{\frac{3}{2}}$

e) $x^{\frac{1}{2}} \times x^{-1}$

f) $y^{\frac{2}{7}} \div y^{\frac{5}{7}}$

g) $\left(\dfrac{x}{y^4}\right)^{\frac{1}{2}}$

h) $\left(\dfrac{x^2}{y}\right)^{-\frac{3}{2}}$