

# Operations on Radicals Lesson #1: Adding and Subtracting Radicals

## Investigation 1

In the last unit we verified that addition of radicals cannot be done by adding the radicands.

In order to develop a rule for adding and subtracting radicals, complete the work below.

- a) Use a calculator to investigate which of the following radical statements are true. Circle the statements which are true and place a single line through the expressions which are false.

i)  $\sqrt{2} + 5\sqrt{2} = 6\sqrt{2}$  ✓

iv)  $7\sqrt{5} + 7\sqrt[3]{5} = 14\sqrt[5]{5}$  ✗

ii)  $4\sqrt[3]{5} - 7\sqrt[3]{5} = -3\sqrt[3]{5}$  ✓

v)  $\sqrt[3]{3} + \sqrt[3]{2} = \sqrt[3]{5}$  ✗

iii)  $5\sqrt{8} - 2\sqrt{8} + 7\sqrt{8} = 10\sqrt{8}$  ✓

- b) Use the results in a) to suggest a rule for adding and subtracting radicals.

① Radicands must be the same  
② index must be the same.

- c) Simplify the following. Express the answer as a mixed radical.

i)  $8\sqrt{7} - 3\sqrt{7} + 15\sqrt{7}$

ii)  $18\sqrt[5]{10} + 12\sqrt[5]{10} - 7\sqrt[5]{10}$

iii)  $5\sqrt{x} - 4\sqrt{x}$

$20\sqrt{7}$

$23\sqrt[5]{10}$

$\sqrt{x}$

## Investigation 2

- a) Use a calculator to verify that the following statements are true.

i)  $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$

ii)  $5\sqrt{12} + 6\sqrt{48} = 34\sqrt{3}$

- b) Does this appear to contradict the rule you wrote in Investigation #1 b)?

- c) Complete the following by writing each radical in simplest mixed form to show that the rule can be modified.

i)  $\sqrt{2} + \sqrt{8}$

ii)  $5\sqrt{12} + 6\sqrt{48}$

$= \sqrt{2} +$

$=$

**Adding and Subtracting Radicals**

In order to add and subtract radicals, they must be able to be expressed as **like radicals**, i.e. radicals with the **SAME radicand** and the **SAME index**.



Write each expression in terms of a single radical.

a)  $\sqrt{80} - \sqrt{20}$

$$\begin{aligned} &\sqrt{16} \sqrt{5} - 2\sqrt{5} \\ &4\sqrt{5} - 2\sqrt{5} \\ &= \boxed{2\sqrt{5}} \end{aligned}$$

b)  $\sqrt[3]{80} + \sqrt[3]{270}$

$$\begin{aligned} &\sqrt[3]{8} \sqrt[3]{10} + 3\sqrt[3]{10} \\ &2\sqrt[3]{10} + 3\sqrt[3]{10} \\ &= \boxed{5\sqrt[3]{10}} \end{aligned}$$

c)  $7\sqrt{27} - 3\sqrt{75} + 2\sqrt{147}$

$$\begin{aligned} &7 \cdot 3\sqrt{3} - 3 \cdot 5\sqrt{3} + 2 \cdot 7\sqrt{3} \\ &21\sqrt{3} - 15\sqrt{3} + 14\sqrt{3} \\ &= \boxed{20\sqrt{3}} \end{aligned}$$



Simplify by combining like radicals.

a)  $-5\sqrt{108} + \frac{3}{4}\sqrt{8} - \frac{5}{4}\sqrt{48} + \frac{1}{2}\sqrt{50}$

$$\begin{aligned} &-5\sqrt{36} \sqrt{3} + \frac{3}{4}\sqrt{4} \sqrt{2} - \frac{5}{4}\sqrt{16} \sqrt{3} + \frac{1}{2}\sqrt{25} \sqrt{2} \\ &-30\sqrt{3} + \frac{3}{2}\sqrt{2} - 5\sqrt{3} + \frac{5}{2}\sqrt{2} \\ &= \boxed{-35\sqrt{3} + 4\sqrt{2}} \\ &\quad \text{or} \\ &\quad \boxed{4\sqrt{2} - 35\sqrt{3}} \end{aligned}$$

b)  $\frac{\sqrt[3]{64}}{8} + 2\sqrt[3]{375} - \frac{2\sqrt[3]{54}}{3} - \frac{5\sqrt[3]{24}}{2}$

$$\begin{aligned} &\frac{1}{8}\sqrt[3]{64} + 2\sqrt[3]{125}\sqrt{3} - \frac{2}{3}\sqrt[3]{27}\sqrt{2} - \frac{5}{2}\sqrt[3]{8}\sqrt{3} \\ &\frac{1}{8}(4) + 10\sqrt{3} - 2\sqrt{2} - 5\sqrt{3} \end{aligned}$$

$$\begin{aligned} &\frac{1}{2} + 10\sqrt{3} - 2\sqrt{2} - 5\sqrt{3} \\ &= \boxed{\frac{1}{2} + 5\sqrt{3} - 2\sqrt{2}} \end{aligned}$$

Complete Assignment Questions #1 - #5

#1-6 (a, c, e...) + 7, 9, 10