Polynomial Functions and Equations Lesson #7:
Investigating the Graphs of Polynomial Functions - Part One

Review of Zeros, Roots, and x-intercepts

Fill in the blanks in the following statement regarding the function with equation $y = P(x)$.

" The zeros of the function, the x-intercepts of the graph of the function, and the roots of the corresponding equation $y = 0$ are the same numbers."

Unique Factorization Theorem

This theorem states that every polynomial function of degree $n \geq 1$ can be written as the product of a leading coefficient, c, and n linear factors to get

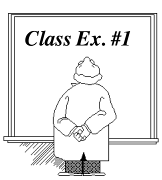
$$P(x) = c(x - a_1)(x - a_2)(x - a_3)\ldots(x - a_n)$$

This theorem implies two ***important*** points for polynomial functions of degree $n \geq 1$:

Point #1: Every polynomial function can be written as a product of its factors and a leading coefficient.

Point #2: Every polynomial function has the same number of factors as its degree. The factors may be real or complex, and may be repeated.

In this lesson we will consider only polynomial functions where the leading coefficient, c, is either 1 or –1. In lesson 10, we will consider polynomial functions with a leading coefficient other than ±1.



The graph of $P(x) = x^3 - 2x^2 - 5x + 6$ is shown. The polynomial has integral zeros.

a) Use the graph to

i) state the zeros of the polynomial

$-2, 1, 3$

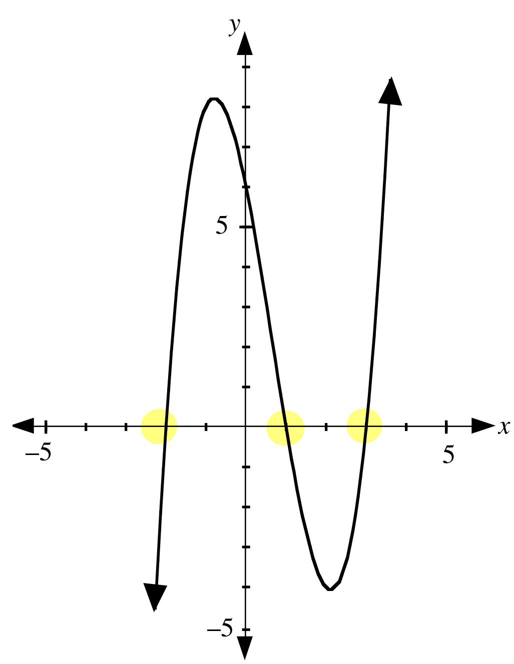
ii) state the factors of the polynomial

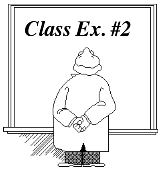
$x+2 \quad x-1 \quad x-3$

iii) write the polynomial in factored form

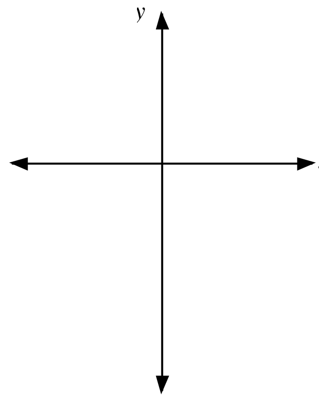
$P(x) = (x+2)(x-1)(x-3)$

b) Use a graphing calculator to sketch $P(x)$ in expanded form and in factored form to verify the above answers.





a) Use a graphing calculator to sketch the graph of the polynomial function $P(x) = -x^3 + 4x^2 + 7x - 10$.



b) Use the graph to state the x-intercepts.

c) Write the polynomial in factored form.

d) Circle the correct alternative:

- The left arm of the graph is (rising / falling).
- The right arm of the graph is (rising / falling).
- The degree of the polynomial is (even / odd).
- The leading coefficient of the polynomial is (positive / negative).

The investigative assignment in this lesson will develop the relationships between the directions of the arms of the graph of a polynomial, the degree of the polynomial, and the sign of the leading coefficient of the polynomial.

Complete Assignment Questions #1 - #3

Assignment

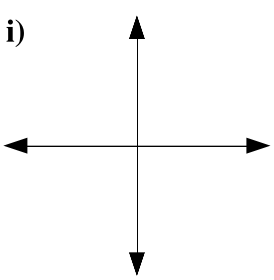
#1 e,f,i, 3

1. In each question use a graphing calculator to:

i) sketch the graph of the polynomial function
ii) state the zeros of the polynomial function
iii) write the polynomial function in factored form.

a) $P(x) = x - 2$

i)

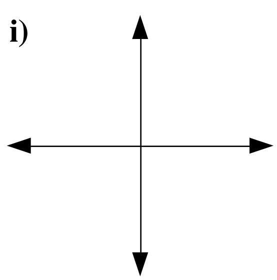


ii)

iii) $P(x) = (x - 2)$

b) $P(x) = -x + 2$

i)

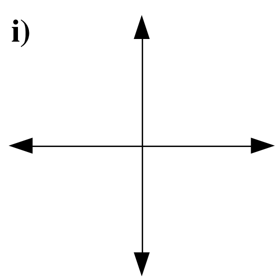


ii)

iii) $P(x) = -(x - 2)$

c) $P(x) = x^2 - 6x + 8$

i)

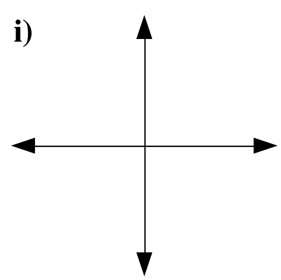


ii)

iii) $P(x) =$

d) $P(x) = -x^2 + 6x - 8$

i)

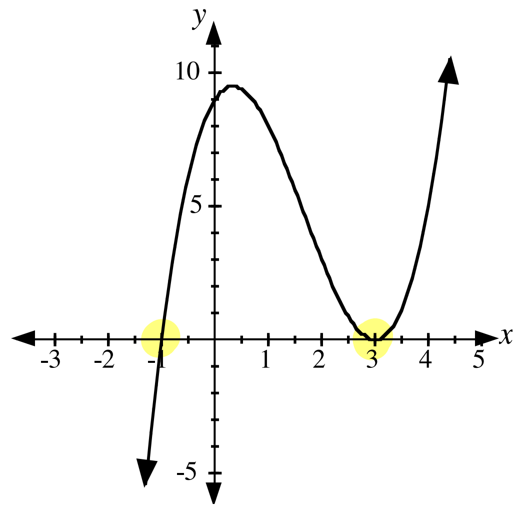


ii)

iii) $P(x) =$

Polynomial Functions and Equations Lesson #8:
Investigating the Graphs of Polynomial Functions - Part Two

Repeated Factors



The graph of the polynomial function
$P(x) = (x + 1)(x - 3)^2$ is shown.

The polynomial has two factors, one of which is repeated. This means that the function has **two distinct** zeros, one of which is a repeated zero.

The factors are $(x - 3)$ which is repeated, and $(x + 1)$.

The zeros of the function are therefore 3, which is repeated, and -1.

The graph of the function has two different x-intercepts, and we say that the function has **two real distinct zeros**, -1 and 3.

- The x-intercept at -1 represents a real zero of the function. passing through
- The x-intercept of 3 represents **two real equal zeros** of the function. tangency.

The repeated zero of 3 is said to be a zero of **multiplicity** 2.
The zero of -1 is a zero of **multiplicity** 1.

Multiplicity

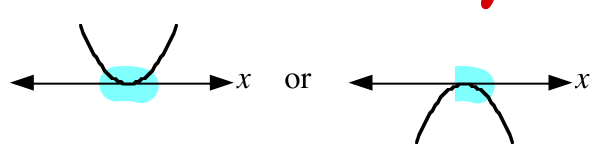
The **multiplicity** of a zero corresponds to the number of times a factor is repeated in the function.

In this lesson, we will investigate how the multiplicity of a zero affects the shape of the graph of a polynomial function. In order to do this, we have to define the following terms.

Tangent

even multiplicity.

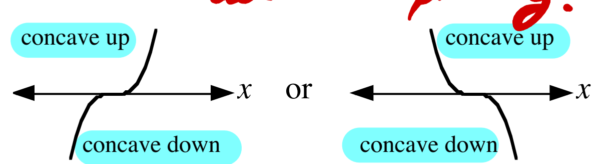
A polynomial graph is **tangent** to the x-axis at a point where the graph **touches** the x-axis and does not cross through it.



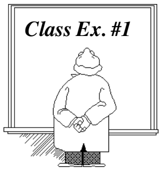
Point of Inflection

odd multiplicity.

A polynomial graph has a **point of inflection** on the x-axis if the graph **changes concavity** at a point on the x-axis.

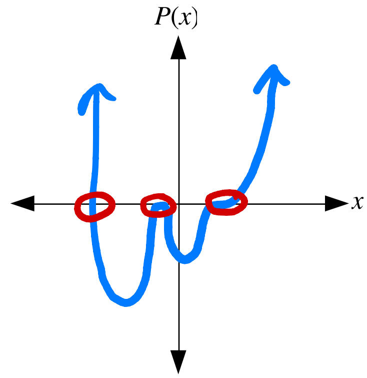


In this lesson we will consider only polynomials where the leading coefficient, c, is either 1 or -1.



Consider the polynomial function

$$P(x) = x^6 - x^5 - 11x^4 + 13x^3 + 26x^2 - 20x - 24$$
$$= (x + 3)(x + 1)^2(x - 2)^3.$$

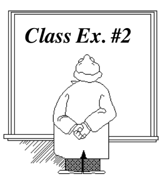


a) Sketch the graph of $P(x)$ using the window
x: [–5, 5, 1] y: [–100, 100, 20]

b) Complete the chart below to state the zeros of $P(x)$, their multiplicities, and whether each zero
- passes straight through the x-axis
- is tangent to the x-axis, or
- has a point of inflection.

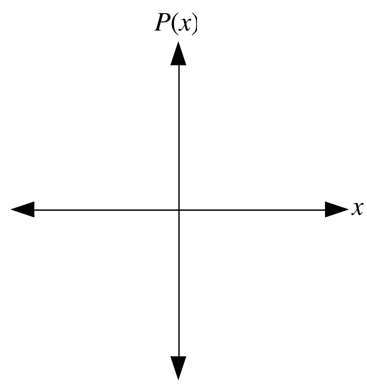
zero	multiplicity	description
-3	1	passes through
-1	2	tangency.
2	3	pt. of inflection.

c) Complete the following.
- The degree of $P(x)$ is 6 .
- The sum of the multiplicities of the zeros of $P(x)$ is 6 .



A polynomial function has the equation

$$P(x) = x^4 + 2x^3 - 15x^2 - 32x - 16.$$



a) Sketch the graph of $P(x)$ using the window
x: [–6, 6, 1] y: [–150, 100, 20]

b) Complete the chart below.

zero	multiplicity	description

c) Complete the following.
- The degree of $P(x)$ is _____ .
- The sum of the multiplicities of the zeros of $P(x)$ is_____ .

d) Write the polynomial in the form $P(x) = (x - a)(x - b)(x - c)^2$.

The investigative assignment in this lesson will develop the relationships between the multiplicities of the zeros of a polynomial function and its graph.

Complete Assignment Questions #1 - #15

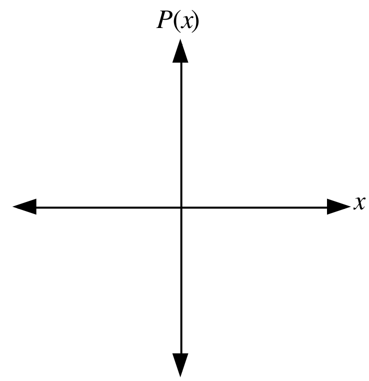
#1, 3

Assignment

In this assignment, choose appropriate windows which will enable you to investigate all the characteristics of the functions.

1. a) Graph $P(x) = x^3 - 4x^2 - 3x + 18$ and complete the table.

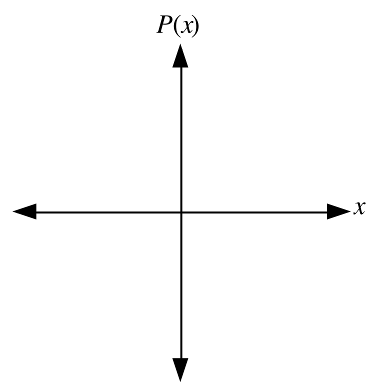
zero	multiplicity



b) Write the polynomial in the form $P(x) = (x - a)(x - b)^2$, where $a, b \in I$.

2. a) Graph $P(x) = x^4 + x^3 - 18x^2 - 52x - 40$ and complete the table.

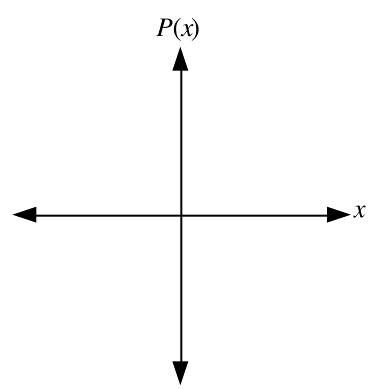
zero	multiplicity



b) Write the polynomial function in factored form.

3. a) Graph $P(x) = -x^3 - 6x^2 + 32$ and complete the table.

zero	multiplicity



b) Write the polynomial in the form $P(x) = -(x - a)(x - b)^2,\ a, b \in I$.

Polynomial Functions and Equations Lesson #9:
Investigating the Graphs of Polynomial Functions - Part Three

In this lesson we will investigate the graphs of polynomial functions which have zeros with multiplicities greater than three.

Investigating Odd Multiplicities

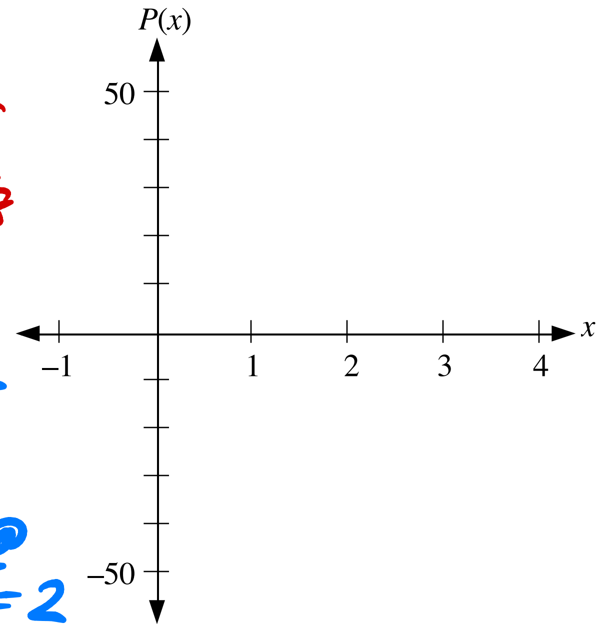
Graph the following functions on the grid showing the x- and y-intercepts.

1. $P(x) = (x - 2)^3$ y-int = −8
2. $P(x) = (x - 2)^5$ y-int = −32
3. $P(x) = (x - 2)^7$ y-int = −128

What happens as the multiplicity of the zero increases through the odd numbers?

curvature is steeper or flatter

point of inflection @ x = 2



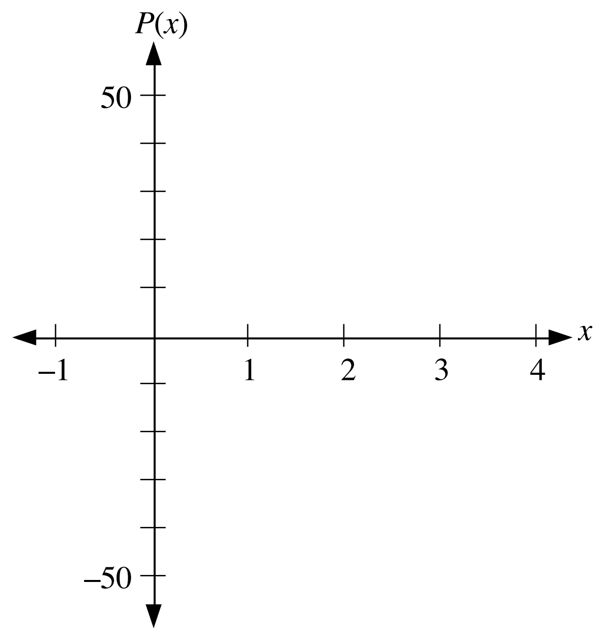
Investigating Even Multiplicities

Graph the following functions on the grid showing the x- and y-intercepts.

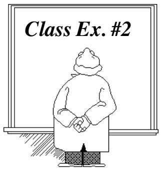
1. $P(x) = (x - 2)^2$
2. $P(x) = (x - 2)^4$
3. $P(x) = (x - 2)^6$

What happens as the multiplicity of the zero increases through the even numbers?

tangency @ x = 2



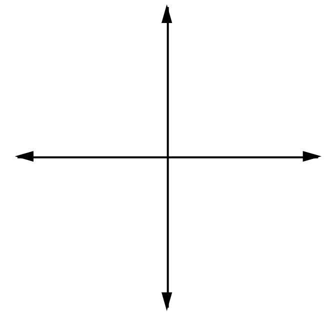
Class Ex. #2



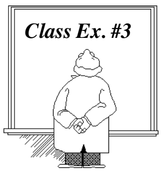
Without using a graphing calculator, make a rough sketch of the graph of

$$f(x) = -x(x-1)^4(x-4).$$

Verify using a graphing calculator.



Class Ex. #3



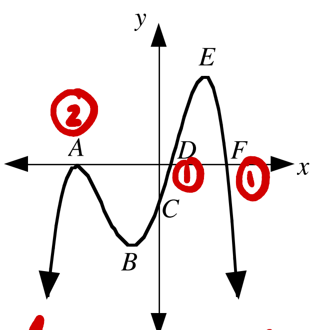
The following graphs represent polynomial functions, $P(x)$, of lowest possible degree.
In each case,

i) state the degree of the polynomial function;

ii) for $P(x) = 0$, state the points on the graph which represent real and equal roots.

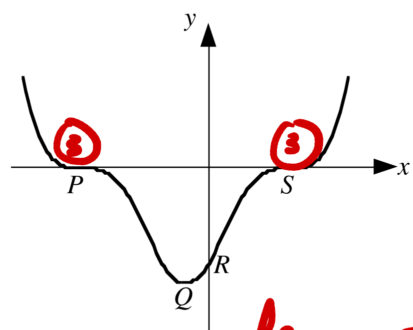
even degree
→ tails down
→ lead coefficient 's negative.

a)



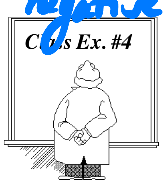
degree = 4
real & equal = A

b)



degree = 6
real & equal = P & S

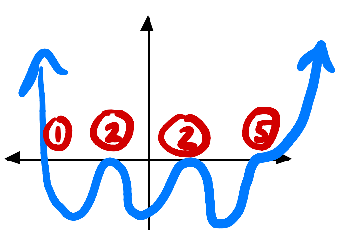
Class Ex. #4



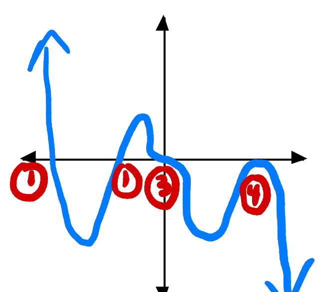
a) On the grid, sketch a graph of a polynomial function satisfying the given conditions.

i)
- positive leading coefficient
- one real zero of multiplicity 1 1
- two real zeros of multiplicity 2 4
- one real zero of multiplicity 5 5

ii)
- negative leading coefficient
- two real zeros of multiplicity 1 1 1
- one real zero of multiplicity 3 3
- one real zero of multiplicity 4 4



degree = 10



degree = 9

b) State the degree of each polynomial function.

Complete Assignment Questions #1 - #5, #8, #9

#1-3, 6, 7