

Class Ex. #2

Express 15° as a difference of two special angles and hence determine the exact value of $\sin 15^\circ$ with a rational denominator.

$$\begin{aligned}\sin 15^\circ &= \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \rightarrow \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}\end{aligned}$$

Class Ex. #3

Express $\frac{5\pi}{12}$ as a sum of two special angles and hence show that $\cot \frac{5\pi}{12} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$.

$$\begin{aligned}\frac{\pi}{2} &= \frac{6\pi}{12} \\ \frac{\pi}{3} &= \frac{4\pi}{12} \\ \frac{\pi}{6} &= \frac{2\pi}{12}\end{aligned}$$

$$\begin{aligned}\frac{5\pi}{12} &= \frac{\pi}{4} + \frac{\pi}{6} \\ \cot \frac{5\pi}{12} &= \frac{1}{\tan \frac{5\pi}{12}}\end{aligned}$$

$$\begin{aligned}\tan \frac{5\pi}{12} &= \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right) \\ &= \tan \frac{\pi}{4} + \tan \frac{\pi}{6} \\ &= \frac{1 - \tan \frac{\pi}{4} \tan \frac{\pi}{6}}{1 + \frac{\tan \frac{\pi}{4}}{\tan \frac{\pi}{6}}} \\ &= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{3}{\sqrt{3}}} = \frac{3+\sqrt{3}}{3-\sqrt{3}}\end{aligned}$$

Class Ex. #4

Simplify the following.

$$\begin{aligned}\text{a)} \sin 100^\circ \cos 10^\circ - \cos 100^\circ \sin 10^\circ &= \sin(A-B) = \sin(100^\circ - 10^\circ) = \sin 90^\circ = \boxed{1} & \cot \frac{5\pi}{12} &= \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{9-6\sqrt{3}+3}{3+3} \\ &= \frac{6}{6} = \boxed{1} & &= \frac{3+3\sqrt{3}}{3+3} = \boxed{1}\end{aligned}$$

$$\begin{aligned}\text{b)} \cos\left(\frac{1}{4}\pi - \theta\right) \cos\left(\frac{1}{4}\pi + \theta\right) - \sin\left(\frac{1}{4}\pi - \theta\right) \sin\left(\frac{1}{4}\pi + \theta\right) &= \cos(A+B) \\ &= \cos\left(\left(\frac{1}{4}\pi - \theta\right) + \left(\frac{1}{4}\pi + \theta\right)\right) = \cos\frac{\pi}{2} = \boxed{0} & &= \frac{6}{6} = \boxed{1}\end{aligned}$$

Class Ex. #5

Given $\cos A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, where $0 \leq A \leq \frac{\pi}{2}$ and $\frac{3\pi}{2} \leq B \leq 2\pi$, find the exact value of $\cos(A+B)$.

$$\begin{aligned}&= \cos A \cos B - \sin A \sin B \\ &= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) \\ &= \frac{15}{65} + \frac{48}{65} \\ &= \boxed{\frac{63}{65}}\end{aligned}$$

LA

$$\begin{aligned}x &= 3 \\ y &= 4 \\ r &= 5\end{aligned}$$

$$\begin{aligned}s^2 &= 3^2 + y^2 \\ 16 &= y^2 \\ 4 &= y\end{aligned}$$

$$\begin{aligned}\sin A &= \frac{4}{5} \\ \sin B &= -\frac{12}{13}\end{aligned}$$

LB

$$\begin{aligned}x &= 5 \\ y &= -12 \\ r &= 13\end{aligned}$$

$$\begin{aligned}13^2 &= s^2 + y^2 \\ 144 &= y^2 \\ 12 &= y\end{aligned}$$

Complete Assignment Questions #1 - #12