Note The identity for $\cos 2A$ can be expressed in two other forms using the Pythagorean identity $\sin^2 A + \cos^2 A = 1$.
The proof of the other two forms of the identity is asked for in assignment question #1.

Double Angle Identities

$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Pythagorean identities.

Class Ex. #1

Prove that the identity $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$ is valid and state the non-permissible values.

$\dfrac{2\tan x}{\sec^2 x}$	$2\sin x \cos x$
$\dfrac{\dfrac{2\sin x}{\cos x}}{\dfrac{1}{\cos^2 x}}$	
$\dfrac{2\sin x}{\cancel{\cos x}} \cdot \dfrac{\cos^{\cancel{2}} x}{1}$	

$2\sin x \cos x = 2\sin x \cos x$

$LS = RS$

Class Ex. #2

a) Use the identity $\sin 2A = 2\sin A \cos A$ with A replaced by $3x$ to determine a double angle identity for $\sin 6x$.

let $3x = A$

$\sin 6x = \sin 2(3x) \longrightarrow \boxed{2\sin 3x \cos 3x}$

b) Write double angle identities for

i) $\cos 10A$

$\cos 2(5A)$

$\cos^2 5A - \sin^2 5A$

ii) $\tan x$

$\tan 2(\tfrac{1}{2}x) \longrightarrow \dfrac{2\tan \frac{1}{2}x}{1-\tan^2(\frac{1}{2}x)}$

Class Ex. #3

Express each of the following in terms of a single trigonometric function.

a) $2\sin 4x \cos 4x$

$\sin 2A = 2\sin A\cos A$

$A = 4x \longrightarrow \sin 2(4x)$

$\boxed{\sin 8x}$

b) $\cos^2 \dfrac{1}{2}A - \sin^2 \dfrac{1}{2}A$

$\cos 2A = \cos^2 A - \sin^2 x$

$A = \tfrac{1}{2}A \rightarrow \cos 2(\tfrac{1}{2}A)$

$= \boxed{\cos A}$

c) $\sin \dfrac{5}{2}x \cos \dfrac{5}{2}x$

$\sin 2A = 2\sin A\cos A$

$A = \tfrac{5}{2}x \rightarrow \tfrac{1}{2}\sin 2(\tfrac{5}{2}x)$

$= \tfrac{1}{2}\boxed{\sin 5x}$

Complete Assignment Questions #1 - #9

#1-6

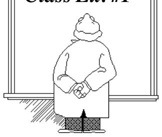
Trigonometry - Equations and Identities Lesson #8:
Using Identities to Solve Equations

We have already learned how to solve simple trigonometric equations.

More complex trigonometric equations may require making substitutions using the trigonometric identities we have learned in this unit. This will usually involve expressing the equation in terms of one of the three primary trigonometric functions.

Using Identities to Solve Equations

Class Ex. #1



Solve the following equations where $0 \le x \le 2\pi$.

a) $2\cos^2 x + 3\sin x = 0$

$2(1-\sin^2 x) + 3\sin x = 0$

$2 - 2\sin^2 x + 3\sin x = 0$

$0 = 2\sin^2 x - 3\sin x - 2$

$0 = (2\sin x + 1)(\sin x - 2)$

$\sin x = -\frac{1}{2}$ $\qquad$ $\cancel{\sin x = 2}$ $\varnothing$

ref∠ $= \frac{\pi}{6}$ in $\frac{\text{III}}{\text{IV}}$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

b) $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$

$\cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6} - \left(\cos x \cos\frac{\pi}{6} + \sin x \sin\frac{\pi}{6}\right)$

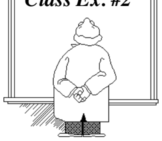
$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = 1$

$-\sin x = 1$

$\sin x = -1$

$\boxed{x = \frac{3\pi}{2}}$

Class Ex. #2



Consider the equation $4 - 7\sin x = \cos 2x$.

a) Which of the three identities for $\cos 2x$ would be the most efficient replacement for solving this equation?

$\cos 2x = 1 - 2\sin^2 x$

b) Determine the general solution to the equation $4 - 7\sin x = \cos 2x$.

$4 - 7\sin x = 1 - 2\sin^2 x$

$2\sin^2 x - 7\sin x + 3 = 0$

$(2\sin x - 1)(\sin x - 3) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\cancel{\sin x = 3}$ $\varnothing$

ref∠ $= \frac{\pi}{6}$ in I & II

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

General Sol'n

$\boxed{\frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n}$

Complete Assignment Questions #1 - #5

$\boxed{\#1, 2, 5, 7, 8}$