Polynomial Functions and Equations Lesson #1:
Polynomial Functions

Overview

In this unit, we will use long division and synthetic division to divide polynomial expressions by binomial expressions, and use these processes as a means to factor polynomial expressions, and to determine the zeros of polynomial functions. We will also establish relationships between the equations of polynomial functions and their graphs.

Polynomial Function

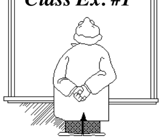
A polynomial function is a function in the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_2x^2 + a_1x + a_0,$$

where: $a_0, a_1, a_2, \dots a_n$ are real numbers, $a_n \neq 0$, and $n \in W$.

- The values, $a_1, a_2, \dots a_n$ are called **coefficients**.
- The coefficient of the highest power of x, which is a_n, is called the **leading coefficient**.
- The term independent of x, which is a_0, is the **constant term**.
- The value of n is the **degree** of the polynomial.

Class Ex. #1



Consider the polynomial $f(x) = x^4 + 7x^3 - 8x^2 + 5$. State:

* descending power order *

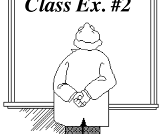
a) the degree of the polynomial **b)** the leading coefficient **c)** the constant term

4 1 5

Recognizing a Polynomial Function

Expressions containing roots of variables, negative or fractional powers of a variable, or any coefficient which is non-real are **NOT** polynomial functions.

Class Ex. #2



State whether or not the following are polynomial functions. If they are not polynomial functions, explain why not.

a) $f(x) = -5x^3 + x^{\frac{1}{2}} - 4$

no, exponent power of $\frac{1}{2}$ on x

b) $f(x) = 2x^2 - 7x^{-1} - 3$

no, negative exponent on $7x$

c) $f(x) = x^4 + 9029x^3 - \sqrt{17}\,x^2 + 3897$

yes.

d) $f(x) = \sqrt{5x^3} - 3x^2 + 2x - 4$

no, $(5x)^{3/2}$

e) $f(x) = 5x^3 - \sqrt{3x^2} + 2x - 4$

$= 5x^3 - \sqrt{3}\,x + 2x - 4$

yes.

f) $f(x) = \sqrt{3}\,x^3 - \sqrt{-3}\,x$

no, $\sqrt{-3}$ is not real #

Degree, Leading Coefficient, and Constant Term

Sometimes a polynomial function can be in a "disguised" form.

Class Ex. #3

State the degree, the leading coefficient, and the constant term of each polynomial function.

$f(x) = -25x^3 + 34x^2 + 2x - 39$

a) $f(x) = 34x^2 - 25x^3 + 2x - 39$

degree→ 3

leading coefficient→ -25

constant→ -39

b) $f(x) = (5x - 1)(2x + 7)$

degree→ 2

leading coefficient→ 10

constant→ -7

Class Ex. #4

In the following polynomial functions,

i) determine the degree, the leading coefficient, and the constant term without expanding
ii) verify the results in i) by expanding and simplifying the polynomial.

a) $P(x) = 2(3x - 1)^2(5x^2 - x + 1)$

degree→ $x^2 \cdot x^2 \rightarrow 4$

leading coefficient→ $2(3)^2(5) = 90$

constant→ $2(-1)^2(1) = 2$

b) $P(x) = 2(3x - 1)^2 + (5x^2 - x + 1)$

degree→ 2

leading coefficient→ $2(3)^2 + 5 = 23$

constant→ $2(-1)^2 + 1 = 3$

Class Ex. #5

Consider the polynomial function $P(x) = 7$.

a) Explain why the polynomial function $P(x) = 7$ has a degree of zero.

$P(x) = 7x^0$ $x^0 = 1$ not necessary

b) Explain why this type of function is called a constant function.

no variable value & the graph is flat horizontal.

Classifying Polynomial Functions

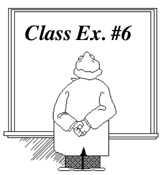
Polynomial functions can be classified in several ways.

By Number of Terms

In previous courses we have used the classification **monomial** (one term), **binomial** (two terms), and **trinomial** (three terms). Polynomials with four or more terms are not usually given a classification other than polynomial.

By Degree

Polynomial functions can also be classified according to degree, such as:
constant (degree zero), **linear** (degree one), **quadratic** (degree two), **cubic** (degree three), **quartic** (degree four), etc. quintic (degree 5)



a) Complete the chart.

Polynomial Function	*Degree*	*Type*
$P(x) = c$	0	constant
$P(x) = ax + b, \quad a \neq 0$	1	linear
$P(x) = ax^2 + bx + c, \quad a \neq 0$	2	quadratic
$P(x) = ax^3 + bx^2 + cx + d, \quad a \neq 0$	3	cubic
$P(x) = ax^4 + bx^3 + cx^2 + dx + e, \quad a \neq 0$	4	quartic

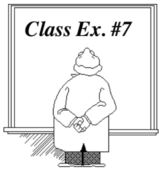
b) Research the names for polynomials of degrees five through ten.

quintic

By Type of Coefficients

Polynomial functions can also be classified according to their coefficients. For example,

- $3x^4 - 5x^2 + x + 7$ is an **integral** polynomial function all coefficients are integers.
- $3x^4 - \frac{2}{5}x^2 + x + 7$ is a **rational** polynomial function at least one rational coefficient
- $\sqrt{3}\,x^4 - \frac{2}{5}x^2 + x + 7$ is a **real** polynomial function all coefficients are real but at least one is irrational.



Class Ex. #7

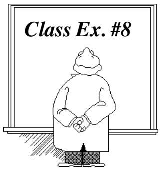
In each case, write a polynomial $P(x)$ satisfying the following conditions.

a) trinomial, quartic, and integral

b) binomial, linear, and real

c) monomial, quadratic, and rational

Evaluating Unknowns in a Polynomial Function



Class Ex. #8

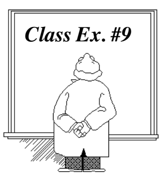
If $P(x) = -3x^2 + ax + 8$ and $P(1) = -9$, then find the value of a.

↳ let $x=1$ & $y=-9$

$P(1) = -3(1)^2 + a(1) + 8$

$-9 = -3 + a + 8$

$\boxed{-14 = a}$



Class Ex. #9

Determine the values of a and b in $P(x) = -2x^2 + ax + b$ if $P(2) = -18$ and $P(-3) = -13$.

$-18 = -2(2)^2 + a(2) + b$

$-18 = -8 + 2a + b$

① $\boxed{-10 = 2a + b}$

system of equations

$-13 = -2(-3)^2 + a(-3) + b$

$-13 = -18 - 3a + b$

② $\boxed{5 = -3a + b}$

elimination

$$\begin{array}{rl} & -10 = 2a + \cancel{b} \\ - & 5 = -3a + \cancel{b} \\ \hline & -15 = 5a \\ & \frac{-15}{5} = \frac{5a}{5} \end{array}$$

$\boxed{a = -3}$

substitute

$5 = 9 + b$

$\boxed{-4 = b}$

$\boxed{\#1\text{-}5(a,c,e\ldots), 6, 7, 14}$

Complete Assignment Questions #1 - #14