

# Polynomial Functions and Equations Lesson #3: Using Synthetic Division to Divide a Polynomial by a Binomial

## Review

The work shows the process of long division used by a student to divide the polynomial  $3x^3 - 7x - 9$  by  $x - 2$ .

$$\begin{array}{r}
 3x^2 + 6x + 5 \\
 x - 2 \overline{) 3x^3 + 0x^2 - 7x - 9} \\
 \underline{3x^3 - 6x^2} \phantom{- 9} \\
 6x^2 - 7x - 9 \\
 \underline{6x^2 - 12x} \phantom{- 9} \\
 5x - 9 \\
 \underline{5x - 10} \\
 1
 \end{array}$$

State

- a) the dividend  $3x^3 - 7x - 9$
- b) the quotient  $3x^2 + 6x + 5$
- c) the divisor  $x - 2$
- d) the remainder  $1$

$$\underline{\underline{3x^3 - 7x - 9 = (x - 2)(3x^2 + 6x + 5) + 1}}$$

## Exploring Synthetic Division

A student who was repeating this course showed his friend a much quicker method for determining the above result. His work is shown.

$$\begin{array}{r}
 \text{placeholder} \\
 2 \quad | \quad \begin{array}{cccc} \textcircled{3} & \textcircled{0} & \textcircled{-7} & \textcircled{-9} \end{array} \\
 \downarrow & & & \\
 3 & \rightarrow & 6 & 12 & 10 \\
 -3 & & 6 & 5 & \textcircled{1}
 \end{array}$$

$$\underline{\underline{3x^3 - 7x - 9 = (x - 2)(3x^2 + 6x + 5) + 1}}$$

a) By looking at both sets of work, explain how the following parts of the synthetic division are related to the long division.

- i) the 2 on the left side *the value that makes the divisor = 0*
- ii) the numbers 3, 0, -7, -9, in the top row *coefficients of the dividend (polynomial)*
- iii) the 1 which is circled *remainder*
- iv) the numbers 3, 6, 5 in the bottom row *coefficients of the quotient (one degree less than started)*

b) Can you identify how the sets of numbers 6, 12, 10, and 3, 6, 5, 1, are obtained?

*diagonal multiplication*      *sum of each column.*

Class Ex. #1



Complete the following synthetic division to determine the quotient and remainder when  $2x^3 - 3x^2 - 8x + 15$  is divided by  $x - 1$ . Express the answer in the form  $\frac{P}{D} = Q + \frac{R}{D}$ .

$$\begin{array}{r|rrrr}
 1 & 2 & -3 & -8 & 15 \\
 & \downarrow & +2 & +(-1) & +(-9) \\
 \hline
 & 2 & -1 & -9 & 6
 \end{array}$$

quotient  $2x^2 - x - 9$  remainder  $6$

$$\frac{2x^3 - 3x^2 - 8x + 15}{x - 1} = 2x^2 - x - 9 + \frac{6}{x - 1}$$

Class Ex. #2



Consider the polynomial  $5x^5 - 6x^4 + 3x^2 - 2x + 1$ .

- a) Use synthetic division to find the quotient and remainder when  $5x^5 - 6x^4 + 3x^2 - 2x + 1$  is divided by  $x + 2$ .

$$\begin{array}{r|rrrrrr}
 -2 & 5 & -6 & 0 & 3 & -2 & 1 \\
 & \downarrow & -10 & 32 & -64 & 122 & -240 \\
 \hline
 & 5 & -16 & 32 & -61 & 120 & -239
 \end{array}$$

quotient:  
 $5x^4 - 16x^3 + 32x^2 - 61x + 120$   
 remainder:  
 $-239$

- b) Find the value of the polynomial when  $x$  is replaced by  $-2$ .

$$\begin{aligned}
 P(-2) &= 5(-2)^5 - 6(-2)^4 + 3(-2)^2 - 2(-2) + 1 \\
 &= -239
 \end{aligned}$$

- c) Comment on your answers in a) and b).

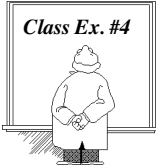
$P(-2)$  is the remainder from synthetic division.

Class Ex. #3



If  $x + 3$  is the divisor in the following synthetic division, calculate the values of  $m$  and  $p$ .

$$\begin{array}{r|rrrr}
 & 2 & 2 & -m & 16 \\
 & & & & n \\
 \hline
 & 2 & & 2m & p
 \end{array}$$



When  $2x^3 - 4x^2 + ax + 3$  is divided by  $x + 2$ , the remainder is 3. Determine the value of  $a$ .

$$\begin{array}{r|rrrr}
 -2 & 2 & -4 & a & 3 \\
 & \downarrow & -4 & 16 & -2a-32 \\
 \hline
 & 2 & -8 & a+16 & -2a-29
 \end{array}$$

remainder = 3

$$\begin{aligned}
 -2a - 29 &= 3 \\
 -32 &= 2a \\
 \frac{-32}{2} &= \frac{2a}{2} \\
 -16 &= a
 \end{aligned}$$

OR

$$P(-2) = 3 = 2(-2)^3 - 4(-2)^2 + (-2)a + 3$$

Complete Assignment Questions #1 - #7

Synthetic Division by  $ax - b$

Use synthetic division to determine the quotient and remainder when the polynomial  $2x^3 + x^2 + 5x - 1$  is divided by  $2x - 1$ .

Note that  $2x - 1 = 2(x - \frac{1}{2})$ .

We divide first by  $x - \frac{1}{2}$ .

$$\begin{array}{r|rrrr}
 \frac{1}{2} & 2 & 1 & 5 & -1 \\
 & \downarrow & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} \\
 \hline
 & 2 & \frac{3}{2} & \frac{13}{2} & \frac{1}{2}
 \end{array}$$

$$P = DQ + R.$$

$$\begin{aligned}
 \text{so } P &= \left(x - \frac{1}{2}\right)(2x^2 + 2x + 6) + 2 \\
 &= \left(x - \frac{1}{2}\right)(2)(x^2 + x + 3) + 2 \\
 &= (2x - 1)(\quad) +
 \end{aligned}$$

Quotient is \_\_\_\_\_

Remainder is \_\_\_\_\_



Divide  $6x^3 - 8x^2 - 5x + 5$  by  $3x + 2$  using synthetic division and write the division in the form  $P = DQ + R$ .

$$\begin{array}{r|rrrr}
 -\frac{2}{3} & 6 & -8 & -5 & 5 \\
 & \downarrow & -4 & 8 & -2 \\
 \hline
 & 6 & -12 & 3 & 3
 \end{array}$$

$3(x + \frac{2}{3})$  (divisor  $3x + 2$ )  
 $6x^2 - 12x + 3$   
 $2x^2 - 4x + 1$  (quotient)  
 $+ 3$  (remainder)

Complete Assignment Questions #8 - #11

#1-11