Functions and Relations Lesson #1:
Functions Review

Overview

In this unit we will develop an understanding of operations on functions and composition of functions. In this lesson we will review some of the properties of polynomial functions, absolute value functions, and radical functions.

Polynomial Functions

A polynomial function in x is a function in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0,$$

where
- $a_0, a_1, a_2, \ldots a_n$ are real numbers, $a_n \neq 0$,
- $n \in W$.

$a_1, a_2, \ldots a_n$ are called <u>coefficients</u>. a_n, is called the <u>leading coefficient</u> and a_0 is the <u>constant term</u>. The value of n is the <u>degree</u> of the polynomial.

descending power order

For example, the polynomial function $f(x) = 7x^3 + x^4 - 8x^2 + 5$ has → $f(x) = x^4 + 7x^3 - 8x^2 + 5$

degree __4__, leading coefficient __1__, and constant term __5__

Three common polynomial functions we will use for transformations are

- Linear Functions
- Quadratic Functions
- Cubic Functions

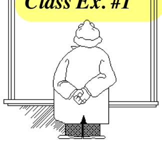
Linear Functions

A linear function is a polynomial function of degree 1 of the form $f(x) = ax + b$. The graph of a linear function is a straight line.

eg. $y = 5$ or $f(x) = 5$

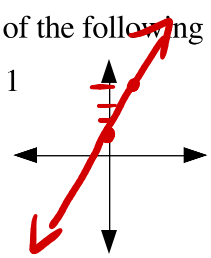
Another function whose graph is a straight line is a **constant function** - a function whose value never changes. It is a polynomial function of degree zero, and can be written in the form $f(x) = ax^0 \Rightarrow f(x) = a$.

Class Ex. #1



Sketch each of the following functions and answer the questions which follow.

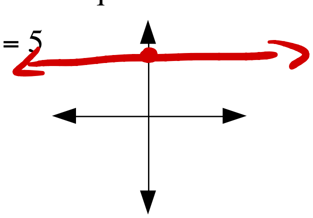
i) $y = 3x + 1$



a) State the domain. $x = \mathbb{R}$

b) State the range. $y = \mathbb{R}$

ii) $y = 5$



a) State the domain. $x = \mathbb{R}$

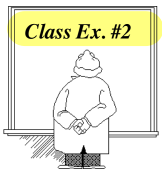
b) State the range. $y = 5$

Quadratic Functions

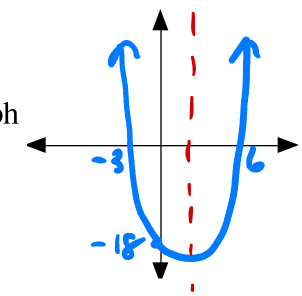
A quadratic function is a polynomial function of degree 2 which can be written in general or standard form.

General Form: $f(x) = ax^2 + bx + c$, where $a \neq 0$.

Standard Form: $f(x) = a(x - p)^2 + q$, where $a \neq 0$.


Class Ex. #2

a) Use a graphing calculator to sketch the graph of the function with equation $y = x^2 - 3x - 18$ and determine



i) the zeros of the function

$x = 6, -3$

ii) the y-intercept of the graph

$y = -18$

iii) the coordinates of the vertex

iv) the domain and range.

D: $x = \mathbb{R}$ R: $y \geq \frac{-81}{4}$

$x = \frac{-3+6}{2} = \frac{3}{2}$

b) Use factoring to determine the zeros.

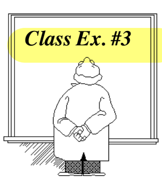
$0 = x^2 - 3x - 18 \longrightarrow 0 = (x-6)(x+3)$

$x = 6, -3$

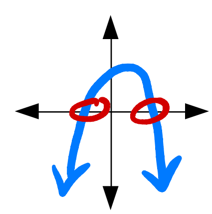
c) Rewrite the equation of the graph of the quadratic function in standard form. Explain how this form helps determine the coordinates of the vertex of the graph.

$x = 3/2$ $y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 18 = \frac{-81}{4}$

vertex $\left(3/2, \frac{-81}{4}\right)$


Class Ex. #3

Use a graphing calculator to sketch the graph of the quadratic function $f(x) = -3x^2 + 4x + 1$.



a) Use the features of a graphing calculator to determine the zeros of the function to the nearest hundredth.

b) Use the quadratic formula to determine the exact values of the zeros in simplest radical form.

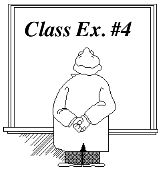
$a = -3$ $b = 4$ $c = 1$

$x = \frac{-4 \pm \sqrt{16 - 4(-3)(1)}}{-6}$

$x = \frac{-4 \pm \sqrt{28}}{-6} = \frac{\cancel{-4}^{2} \pm \cancel{2}^{1}\sqrt{7}}{\cancel{-6}_{3}} = \boxed{\frac{2 \pm \sqrt{7}}{3}}$

Cubic Functions

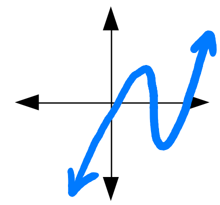
A cubic function is a polynomial function of degree 3 of the form
$f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$.



Class Ex. #4

Consider the cubic function with equation $y = x^3 - 8x^2 + 16x - 8$.

a) Use a graphing calculator to sketch the graph of the cubic function.



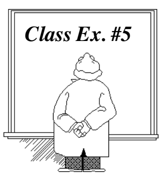
b) Use the features of a graphing calculator to determine the x-intercepts of the graph to the nearest tenth.

$\boxed{x = 0.8, 2.0, 5.2}$

Absolute Value Function

An absolute value function is a function of the form $f(x) = |x|$, where

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

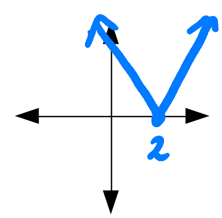


Class Ex. #5

Sketch the graph of the absolute value function $f(x) = |x - 2|$ and determine the domain and range.

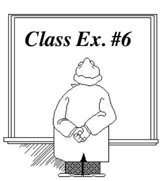
$x \to x - 2$
h.t. 2 units right

$D = x = \mathbb{R}$
$R = y \geq 0$



Radical Functions

A radical function is a function which contains a variable in the radicand such as $f(x) = \sqrt{x}$.



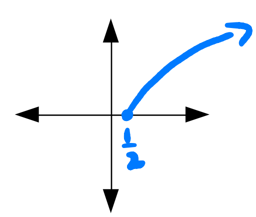
Class Ex. #6

a) Sketch the radical function $g(x) = \sqrt{2x - 1}$.

$x \to 2x - 1$
$2(x - \frac{1}{2})$

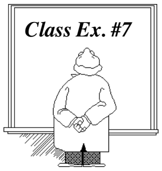
b) Determine the domain and range of the function.

$D: x \geq \frac{1}{2}$
$R: y \geq 0$



Complete Assignment Questions #1 - #8

Function Notation



If $f(x) = x^2 + 4x + 5$, find the following in simplest form:

a) $f(2)$ $= 2^2 + 4(2) + 5$ $= \boxed{17}$

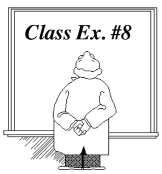
b) $2f(x)$ $= 2(x^2 + 4x + 5)$ $= \boxed{2x^2 + 8x + 10}$

c) $f(2x)$ $= (2x)^2 + 4(2x) + 5$ $= \boxed{4x^2 + 8x + 5}$

d) $f(x) - 2$ $= (x^2 + 4x + 5) - 2$ $= \boxed{x^2 + 4x + 3}$

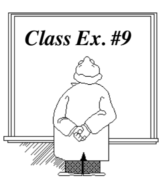
e) $f(x - 2)$ $= (x-2)^2 + 4(x-2) + 5$ $= x^2 - 4x + 4 + 4x - 8 + 5$ $= \boxed{x^2 + 1}$

f) $f(-x)$ $= (-x)^2 + 4(-x) + 5$ $= \boxed{x^2 - 4x + 5}$



Consider the function $f(x) = x^3$. Match each function on the right with an expression on the left.

Expression	**Function**
A. $x^3 + 5$	**1.** $f(x + 5)$
B. $(x + 5)^3$	**2.** $f(5x)$
C. 5^3	**3.** $f(x) + 5$
D. $(5x)^3$	**4.** $f(5)$



If $f(x) = |x|$, write the following in terms of the function f.

a) $|x| - 4$ $f(x) - 4$

b) $|x - 4|$ $f(x - 4)$

c) $2|x|$ $2f(x)$

d) $|2x|$ $f(2x)$

e) $3 - |x|$ $3 - f(x)$

f) $|x + 3| - 2$ $f(x+3) - 2$

g) $\dfrac{1}{|x|}$ $\dfrac{1}{f(x)}$

h) $|-x|$ $f(-x)$

Complete Assignment Questions #9 - #14

#1, 3, 5, 10

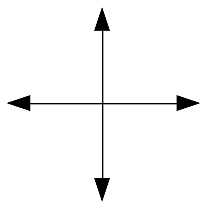
Assignment

1. The equations of the graphs of five functions are given below.
In each case, sketch the graph and determine

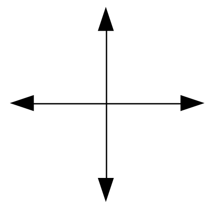
i) the zeros (to the nearest tenth if necessary)

ii) the domain and range.

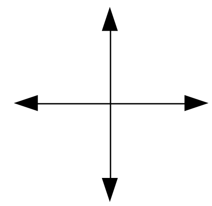
a) $y = -2x + 4$



b) $y = 5x - 10$



c) $y = x^2 - 2x - 15$

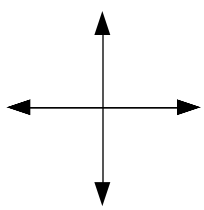


zeros

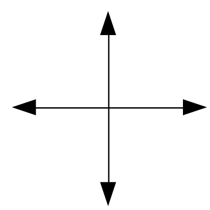
domain

range

d) $y = -x^2 - 3x - 4$



e) $y = (x + 1)(x + 2)(x - 5)$



zeros

domain

range

2. The linear function with equation $y = ax + b$ is in slope y-intercept form.

a) Which parameter represents the slope of the line?

b) Which parameter represents the y-intercept?

c) If $a > 0$, describe the slope.

d) If $a < 0$, describe the slope.

3. How can you tell from the quadratic function $f(x) = ax^2 + bx + c$ whether the graph of the function will open up or open down?

Functions and Relations Lesson #2:
Operations with Functions - Part One

Operations with Functions

The following properties apply to functions f and g, provided x is in the domain of f and g.

The <u>sum</u> of f and g	$\rightarrow$	$(f+g)(x) = f(x) + g(x)$
The <u>difference</u> of f and g	$\rightarrow$	$(f-g)(x) = f(x) - g(x)$
The <u>product</u> of f and g	$\rightarrow$	$(fg)(x) = f(x)g(x)$
The <u>quotient</u> of f and g	$\rightarrow$	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

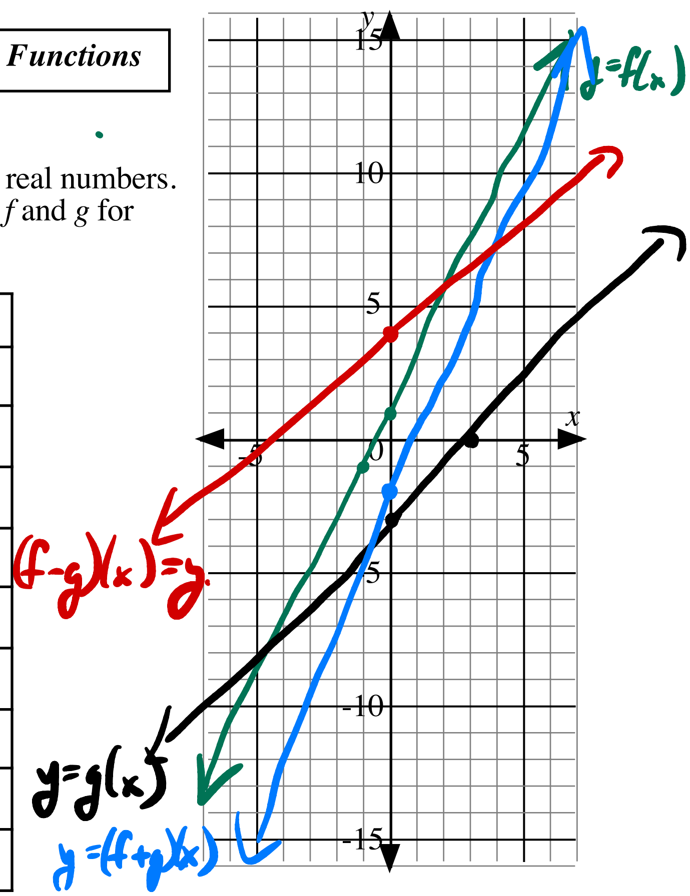
In this lesson, we will consider the sum and difference of two functions, and in the next two lessons we will consider the product and quotient of two functions.

Investigating the Sum & Difference of Two Functions

<u>Part 1</u>:

Two functions $f(x)$ and $g(x)$ are defined for all real numbers. The table of values below shows the values of f and g for certain values of x.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$
–4	–7	–7	−14	0
–3	–5	–6	−11	1
–2	–3	–5	−8	2
–1	–1	–4	−5	3
0	1	–3	−2	4
1	3	–2	1	5
2	5	–1	4	6
3	7	0	7	7
4	9	1	10	8



a) Plot the points $(x, f(x))$ on the grid and sketch the graph of $y = f(x)$ for $x \in R$.

b) Plot the points $(x, g(x))$ on the grid and sketch the graph of $y = g(x)$ for $x \in R$.

c) Complete the tables above for $(f+g)(x)$ and $(f-g)(x)$ and sketch the graphs of $y = (f+g)(x)$ and $y = (f-g)(x)$ for $x \in R$.

d) The functions f and g on the previous page are $f(x) = 2x + 1$ and $g(x) = x - 3$. Write expressions for the functions $(f + g)(x)$ and $(f - g)(x)$.

$(f+g)(x) = (2x+1)+(x-3) = \boxed{3x-2}$

$(f-g)(x) = (2x+1)-(x-3) = 2x+1-x+3 = \boxed{x+4}$

e) Use a graphing calculator to graph the functions $y = (f + g)(x)$ and $y = (f - g)(x)$ from part d) and compare these graphs with the graphs from part c).

f) State the domains and ranges of the functions $(f + g)(x)$ and $(f - g)(x)$.

D: $x = \mathbb{R}$ R: $y = \mathbb{R}$ for both.

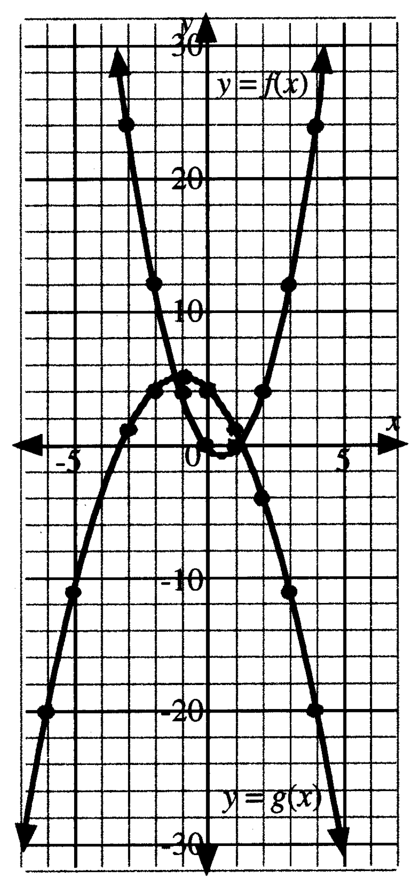
g) Determine the values of $(f + g)(10)$ and $y = (f - g)(-10)$.

$3(10)-2 = \boxed{28}$ $(-10)+4 = \boxed{-6}$

h) In this example, the sum and difference of two functions of degree 1 are also functions of degree 1. Can you find two functions of degree 1 whose sum or difference is not a function of degree 1?

Part 2:

The graphs of two quadratic functions, $y = f(x)$ and $y = g(x)$, $x \in R$, are shown on the grid. The points marked with dots have integer coordinates.



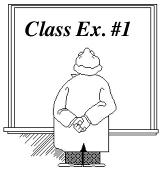
a) Use these graphs to sketch the graphs of $y = (f + g)(x)$ and $y = (f - g)(x)$ for $x \in R$.

b) The functions f and g above are $f(x) = 2x^2 - 2x$ and $g(x) = -x^2 - 2x + 4$. Write expressions for the functions $(f + g)(x)$ and $(f - g)(x)$.

c) Use a graphing calculator to graph the functions $y = (f + g)(x)$ and $y = (f - g)(x)$ from part b) and compare these graphs with the graphs from part a).

d) State the domains of the functions $(f + g)(x)$ and $(f - g)(x)$.

e) In this example, the sum and difference of two quadratic functions are quadratic functions. Can you find two quadratic functions whose sum or difference is not a quadratic function?



Consider the functions $f(x) = x^3 - 2x^2 + 6$ and $g(x) = x^3 - 2x + 6$, $x \in R$.

a) Determine expressions for

i) $f(x) + g(x)$

$(x^3-2x^2+6)+(x^3-2x+6)$

$= 2x^3-2x^2-2x+12$

ii) $f(x) - g(x)$

$(x^3-2x^2+6)-(x^3-2x+6)$

$= -2x^2+2x$

b) Complete the following statements

i) $f(x) + g(x)$ is a sum of two functions f and g, and can be rewritten as $(f + g)(x) =$ $2x^3-2x^2-2x+12$.

ii) $f(x) - g(x)$ is a difference of two functions f and g, and can be rewritten as $(f-g)(x)$ = $-2x^2+2x$

c) Evaluate

i) $(f + g)(-2)$

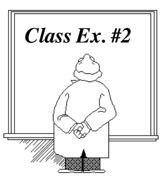
$= 2(-2)^3-2(-2)^2-2(-2)+12$

$= \boxed{-8}$

ii) $(f - g)(\sqrt{6})$

$= -2(\sqrt{6})^2+2(\sqrt{6})$

$= \boxed{-12+2\sqrt{6}}$



A multi-media production company produces DVDs which cost \$3.00 per unit. The fixed costs, including the graphics, are \$10 000 irrespective of the number of DVDs produced. Each DVD retails for \$15.00.

If x is the number of units produced, answer the following in terms of x.

a) Write the total cost, $C(x)$, as a function of the number of units produced.

b) Write the revenue, $R(x)$, as a function of the number of units produced.

c) Write the company's profit, $P(x)$, as a function of the number of units produced.

d) Determine the company's profit if 20 000 DVDs are produced

Complete Assignment Questions #1 - #9

$\boxed{\#1-3}$