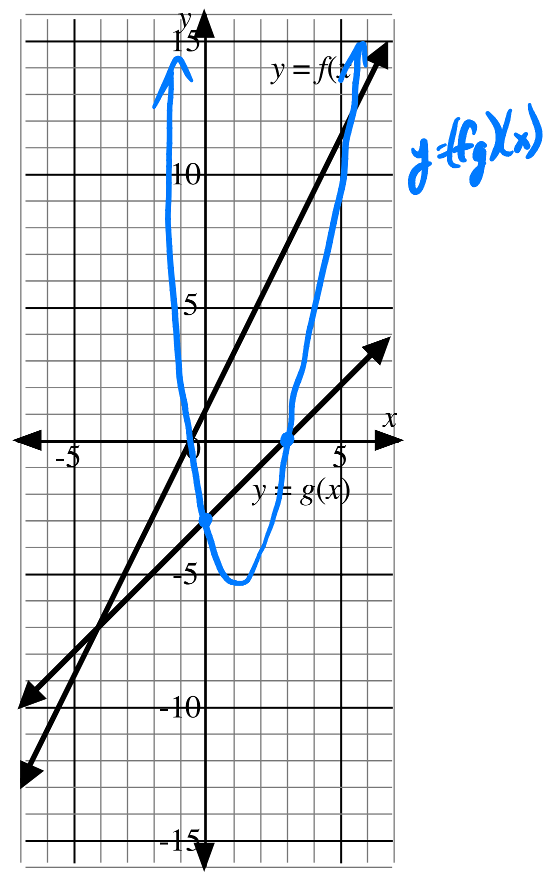
Functions and Relations Lesson #3:
Operations with Functions - Part Two

Investigating the Product of Two Functions

In this investigation we will use the same functions, f and g, as in the investigation in lesson 2.

x	$f(x)$	$g(x)$	$(fg)(x)$
–4	–7	–7	49
–3	–5	–6	30
–2	–3	–5	15
–1	–1	–4	4
0	1	–3	-3
1	3	–2	-6
2	5	–1	-5
3	7	0	0
4	9	1	9



a) Complete the table above for $(fg)(x)$.

b) Plot the points from the table which will fit on the grid and complete the sketch of $y = (fg)(x)$ for $x \in R$.

c) The functions f and g above are $f(x) = 2x + 1$ and $g(x) = x - 3$.
Write a simplified expression for the function $(fg)(x)$.

$(fg)(x) = (2x+1)(x-3)$
$= 2x^2 - 5x - 3$

d) Use a graphing calculator to graph the function $y = (fg)(x)$ from part c) and compare this graph with the graph from part b).

e) Determine the domain and range of the function $(fg)(x)$.

D: $x = \mathbb{R}$ R: $y \geq -\frac{49}{8}$

f) Calculate the value of $(fg)(8)$ in two different ways.

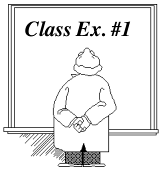
$(fg)(8) = 2(8)^2 - 5(8) - 3$
$= \boxed{85}$

$f(8) \cdot g(8) = (17)(5)$
$= \boxed{85}$

g) Complete the statement.
" In this investigation, the product of two functions whose graphs are straight lines is a quadratic function."

Can you find two functions whose graphs are straight lines where this is not the case?

? $f(x) = 5$



Consider the functions $f(x) = 3\sqrt{x} - 2$ and $g(x) = \sqrt{x} - 5$.

a) Write an expression in simplest form for each of the following functions.

i) $(f - g)(x)$

$= (3\sqrt{x} - 2) - (\sqrt{x} - 5)$

$= \boxed{2\sqrt{x} + 3}$

ii) $(fg)(x)$

$= (3\sqrt{x} - 2)(\sqrt{x} - 5)$

$= \boxed{3x - 17\sqrt{x} + 10}$

b) Evaluate

i) $(f - g)(16)$

$= 2\sqrt{16} + 3$

$= \boxed{11}$

ii) $(fg)(49)$

$= 3(49) - 17\sqrt{49} + 10$

$= \boxed{38}$

Complete Assignment Questions #1 - #6

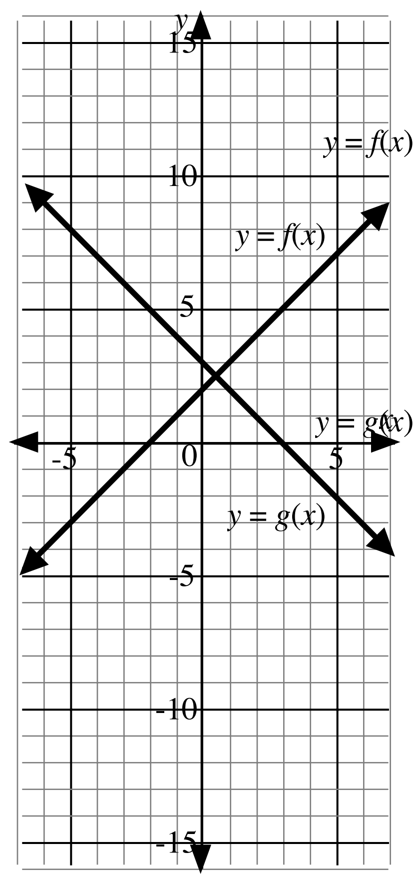
Assignment

$\boxed{\#1, 4, 6}$

1. Consider functions f and g defined for all real numbers. Partial graphs of the functions are shown on the grid. Both functions have integer values when x is an integer.

a) Complete the table above for $(fg)(x)$.

x	$f(x)$	$g(x)$	$(fg)(x)$
–4			
–3			
–2			
–1			
0			
1			
2			
3			
4			
5			



b) Plot the points from the table which will fit on the grid and complete the sketch of $y = (fg)(x)$ for $x \in R$.

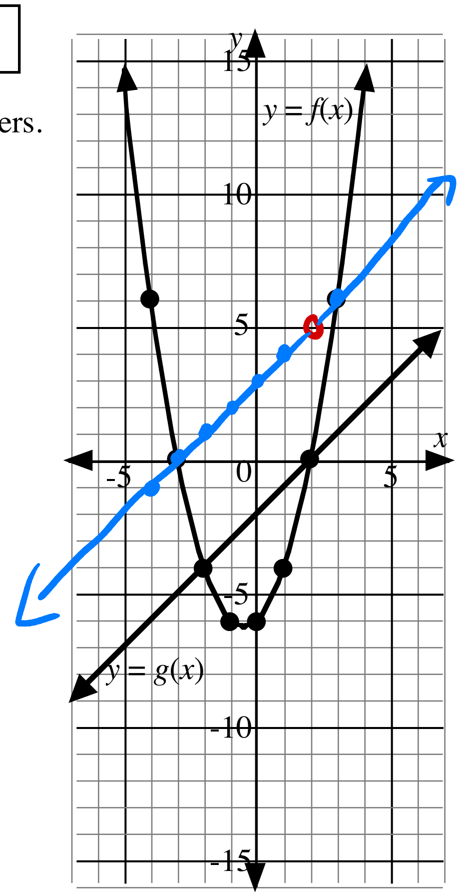
Functions and Relations Lesson #4:
Operations with Functions - Part Three

Investigating the Quotient of Two Functions

Two functions $f(x)$ and $g(x)$ are defined for all real numbers.
The graphs of the functions are shown on the grid.
The points marked with dots have integer coordinates.

a) Complete the table below.

x	$f(x)$	$g(x)$	$\left(\frac{f}{g}\right)(x)$
4	14	2	14/2 = 7
3	6	1	6
2	0	0	undefined
1	−4	−1	4
0	−6	−2	3
−1	−6	−3	2
−2	−4	−4	1
−3	0	−5	0
−4	6	−6	−1



$y = \left(\frac{f}{g}\right)(x)$

b) Explain why the domain of the function $\left(\frac{f}{g}\right)(x)$ is not $x \in R$.

$g(x) = 0$ there is undefined ie. when $x = 2$

D: $x = \mathbb{R}$ except $x \neq 2$

c) Sketch the graph of $y = \left(\frac{f}{g}\right)(x)$ showing the domain restriction by drawing an open circle on the graph.

point of discontinuity.

d) The functions f and g above are $f(x) = x^2 + x - 6$ and $g(x) = x - 2$. Write and simplify an expression for the function $\left(\frac{f}{g}\right)(x)$, including the domain restriction.

$$\left(\frac{f}{g}\right)(x) = \frac{x^2+x-6}{x-2} = \frac{\cancel{(x-2)}(x+3)}{\cancel{x-2}} = \boxed{x+3}$$

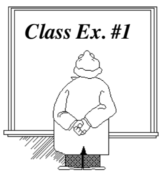
$x \neq 2$

point discontinuity (2, 5)

e) Evaluate $\left(\frac{f}{g}\right)(-6)$. $= -6 + 3 = \boxed{-3}$

- In the Investigation, the graph of $\left(\frac{f}{g}\right)(x)$ cannot include the point $(2, 5)$ since there is a domain restriction $x \neq 2$. This results in a "hole" in the graph represented by an open circle. We refer to this hole as a **point of discontinuity** and this concept will be investigated in more detail in the unit on Rational Functions.

- In the Investigation, the quotient of a quadratic function and a linear function simplifies to a linear function with a domain restriction. This is not always the case. In many examples the quotient of two functions cannot be reduced to a simpler function.



Class Ex. #1

In each case, write and simplify (where possible) an expression for $\left(\frac{f}{g}\right)(x)$.

Include any domain restrictions.

a) $f(x) = 2x + 5,\ g(x) = x + 5$

$$\left(\frac{f}{g}\right)(x) = \frac{2x+5}{x+5} \qquad x \neq -5$$

b) $f(x) = x^2 - 2x - 35,\ g(x) = x - 7$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2-2x-35}{x-7} \qquad x \neq 7$$

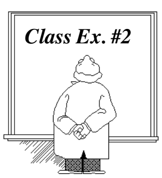
$$= \frac{\cancel{(x-7)}(x+5)}{\cancel{x-7}}$$

$$= x+5$$

c) $f(x) = x - 2,\ g(x) = x^2 - 5x + 6$ $\quad x \neq 2, 3$

$$\left(\frac{f}{g}\right)(x) = \frac{x-2}{x^2-5x+6} = \frac{\cancel{x-2}}{(x-3)\cancel{(x-2)}}$$

$$= \frac{1}{x-3}$$



Class Ex. #2

Consider the functions $f(x) = 2x^2 - 13x - 7$ and $g(x) = 2x + 1$.

a) State the domains of f and g.

$x = \mathbb{R}$ for both.

b) Write an expression in simplest form for $\left(\frac{f}{g}\right)(x)$. State the domain.

$$\left(\frac{f}{g}\right)(x) = \frac{2x^2-13x-7}{2x+1}$$

$$= \frac{\cancel{(2x+1)}(x-7)}{\cancel{2x+1}}$$

$$= x-7$$

$x = \mathbb{R}$ except $x \neq -\frac{1}{2}$

c) **Explain** two different ways to evaluate $\left(\frac{f}{g}\right)(3)$. Calculate $\left(\frac{f}{g}\right)(3)$.

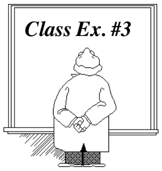
① $= 3-7$

$= \boxed{-4}$

② $\dfrac{2(3)^2-13(3)-7}{2(3)+1}$

$= \boxed{-4}$

Complete Assignment Questions #1 - #7



Consider the functions $f(x) = \dfrac{2x}{x-1}$ and $g(x) = \dfrac{x}{x-3}$.

a) State the domains of f and g.

$f(x)$: $x = \mathbb{R}$ except $x \neq 1$ $\quad$ $g(x)$: $x = \mathbb{R}$ except $x \neq 3$

b) Evaluate $3(fg)(2)$.

$$3\left(\frac{2(2)}{2-1}\right)\left(\frac{2}{2-3}\right) = \boxed{-24}$$

c) Write an expression in simplest form for $(f+g)(x)$. State any restrictions on x.

$$(f+g)(x) = \frac{2x}{x-1}^{(x-3)} + \frac{x}{x-3}^{(x-1)} = \frac{2x^2-6x+x^2-x}{(x-1)(x-3)}$$

$$= \frac{3x^2-7x}{(x-1)(x-3)} = \frac{x(3x-7)}{(x-1)(x-3)}$$

$x \neq 1, 3$

d) Write an expression in simplest form for $\left(\dfrac{f}{g}\right)(x)$. State any restrictions on x.

$\boxed{x \neq 1, 3, 0}$

$$\left(\frac{f}{g}\right)(x) = \frac{2x}{x-1} \div \frac{x}{x-3}$$

$$= \frac{2\cancel{x}^{1}}{x-1} \cdot \frac{x-3}{\cancel{x}_{1}}$$

$$= \frac{2(x-3)}{x-1} \text{ or } \frac{2x-6}{x-1}$$

Complete Assignment Questions #8 - #11

$\boxed{\# 1, 3\text{-}5, 8, 9}$