The function $h(x) = 2x + 3$ is a composition of two functions $f(x) = 2x$ and $g(x) = x + 3$.

The **composite function** $h(x)$ can be written in the form:

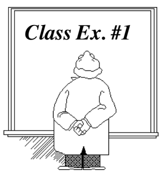
$\boldsymbol{h(x) = g(f(x))}$ read as "g of f of x"

or

$\boldsymbol{h(x) = (g \circ f)(x)}$.

gof's fog's

- When $h(x)$ is written as $g(f(x))$, note that function f is applied first and the function g is applied second.
- h is often referred to as a function of a function.
- The techniques used in Class Ex. #1 and Warm-Up #4 will benefit students who plan to study calculus (the Chain Rule) in future years.



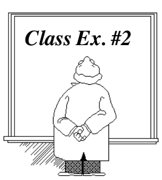
Consider the composite function $h(x) = x^2 - 2$.

a) Describe in words, two operations, in order, which can be applied to x to end up with $x^2 - 2$.

b) Complete the diagram and write $h(x)$ as a composition of two functions, f and g, where $h(x) = g(f(x))$.

$$x \xrightarrow[\text{square}]{f} \boxed{} \xrightarrow[\text{subtract 2}]{g} \boxed{}$$

$f(x) =$ ______ $g(x^2) =$ ______ so $g(x) =$ ______



Consider the composite function $h(x) = (x + 4)^3$.

a) Describe in words, two operations, in order, which can be applied to x to end up with $(x + 4)^3$

b) Complete the diagram and write $h(x)$ as a composition of two functions, f and g, where $h(x) = g(f(x))$.

$$x \xrightarrow[\text{\_\_\_\_\_\_\_\_}]{f} \boxed{} \xrightarrow[\text{\_\_\_\_\_\_\_\_}]{g} \boxed{}$$

$f(x) =$ ______ $g(\quad) =$ ______ so $g(x) =$ ______

Developing a Method for the Composition of Two Functions

Consider two functions $f(x) = 3x$ and $g(x) = x - 5$.

a) Complete the diagram to determine a formula for the composite function $h(x) = g(f(x))$.

$x \xrightarrow[\text{multiply by 3}]{f} \boxed{} \xrightarrow[\text{subtract 5}]{g} \boxed{}$

$h(x) = g(f(x)) = g(\_\_\_\_\_\_) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$

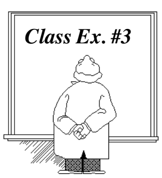
b) Use a similar technique to determine a formula for the composite function $k(x) = f(g(x))$.

Complete Assignment Questions #1 - #6

Composition of Functions

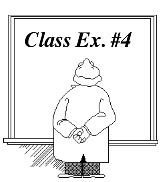
Consider the composite function $g(f(x)) = (g \circ f)(x)$ where $f(x)$ and $g(x)$ are given.
Use the following procedure to determine $g(f(x))$.

Start with $g(f(x))$. → Replace $f(x)$ with the formula for f. → Apply the formula for g. → Write the answer in simplest form.



Given $f(x) = 10x + 1$ and $g(x) = 2x - 5$, complete the work below to determine $(g \circ f)(x)$.

STEPS	WORK
Step 1: Start with $g(f(x))$	Step 1: $g(f(x))$
Step 2: Replace $f(x)$ with the formula for f.	Step 2:
Step 3: Apply the formula for g.	Step 3:
Step 4: Write the answer in simplest form.	Step 4:



If $f(x) = \boxed{2x^2 - 1}$ and $g(x) = 3x - 4$, find

a) $(g \circ f)(x)$

$= 3(2x^2 - 1) - 4$
$= 6x^2 - 3 - 4$
$= \underline{6x^2 - 7}$

b) $(g \circ g)(x)$

$= 3(3x - 4) - 4$
$= 9x - 12 - 4$
$= \underline{9x - 16}$

Domain and Range of a Composite Function

Consider the functions $f(x) = x^2 - 3$ and $g(x) = \sqrt{x-1}$.

a) Without finding a formula for $(f \circ g)(x)$ or $(g \circ f)(x)$, evaluate:

i) $(f \circ g)(2)$

$g(2) = \sqrt{2-1}$
$= 1$
$f(1) = 1-3$
$= \boxed{-2}$

ii) $(f \circ g)(0)$

$g(0) = \sqrt{0-1}$
$\boxed{\varnothing}$

iii) $(g \circ f)(2)$

$f(2) = 2^2 - 3$
$= 1$
$g(1) = \sqrt{1-1}$
$= \boxed{0}$

iv) $(g \circ f)(0)$

$f(0) = 0-3$
$= -3$
$g(-3) = \boxed{\varnothing}$

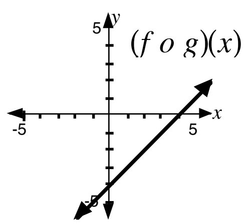
b) State the domains of f and g.

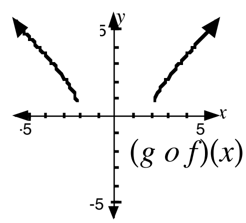
$f(x)$: D: $x \in \mathbb{R}$
$g(x)$: D: $x \geq 1$

c) State the ranges of f and g.

$f(x)$: R: $y \geq -3$
$g(x)$: R: $y \geq 0$

d) In order to determine the domain and range of $f \circ g$ and $g \circ f$, Aaron found expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$ and used his graphing calculator to sketch the graphs of the composite functions. He obtained the following graphs.





He concluded that

domain of $f \circ g = \{x \mid x \in R\}$
range of $f \circ g = \{y \mid y \in R\}$

domain of $g \circ f = \{x \mid x \leq -2 \text{ or } x \geq 2, x \in R\}$
range of $g \circ f = \{y \mid y \geq 1, y \in R\}$

There are errors in Aaron's thinking.

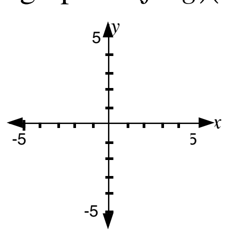
i) Find expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$.

$(f \circ g)(x) = (\sqrt{x-1})^2 - 3$
$= x - 1 - 3$
$= \boxed{x-4}$

$(g \circ f)(x) = \sqrt{(x^2-3)-1}$
$= \boxed{\sqrt{x^2-4}}$

ii) Use the expressions in **d) i)** and the results from **a)**, **b)**, and **c)** to complete the following:

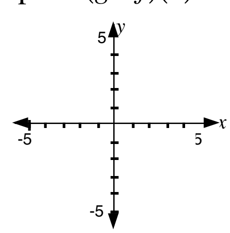
graph of $(f \circ g)(x)$



domain of $f \circ g =$ $x \geq 1$

range of $f \circ g =$ $y \geq -3$

graph of $(g \circ f)(x)$



domain of $g \circ f =$ $x \leq -2, x \geq 2$

range of $g \circ f =$ $y \geq 0$

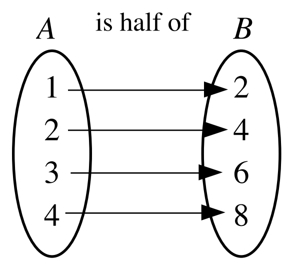
$\boxed{\#7\text{–}12}$

Functions and Relations Lesson #6:
The Inverse of a Relation - Part One

In this lesson we will work with relations that are functions.

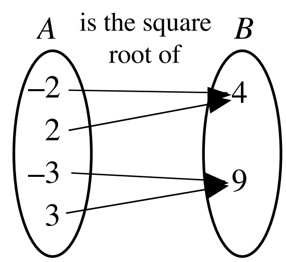
A **function** is a relation in which each element of a set A (the domain) is mapped to one and only one element of a set B (the range).

eg. **i)**



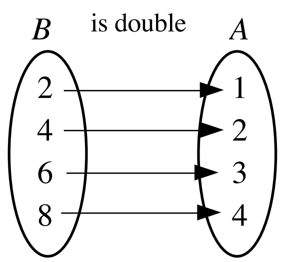
or

ii)



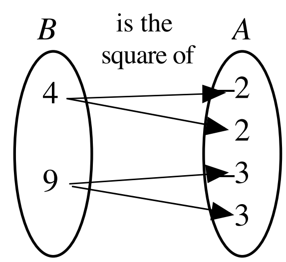
The **inverse of a function** is a relation which "undoes" what the function does. In other words, the elements in set B are mapped back to elements in set A.

i)



or

ii)



Referring to the cases above, complete the following by choosing the correct answer.

In case (i) the inverse (is / is not) a function.

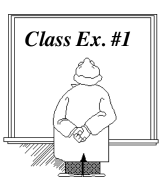
In case (ii) the inverse (is / is not) a function.

- The **domain of the inverse** is the **range of the original** function.
- The **range of the inverse** is the **domain of the original** function.
- **The inverse of a function may or may not be a function**.

one "x" in...
one "y" out.

$f^{-1}(x)$

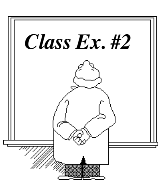
____



Consider the "operation" of putting on your socks and then putting on your shoes. What would be the "inverse operation"?

____

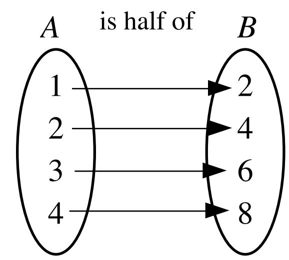
Finding the Inverse of a Function Defined in Words



Complete the table to describe the inverse of the function:

FUNCTION	INVERSE	Is the inverse a function?
multiply by 2	________________	_____
square	________________	_____
take the reciprocal	________________	_____
divide by 3, then add 1	________________	_____

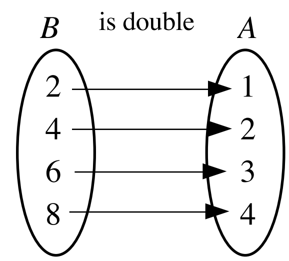
Finding the Inverse of a Function Defined by Ordered Pairs



Consider the arrow diagram on the previous page.

The function which maps from A to B can be described by the following set of ordered pairs:

{(1, 2), (



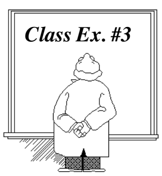
The inverse function which maps from B to A can be described by the following set of ordered pairs:

{(2, 1),

$(x,y) \to (y,x)$

Notice that the ordered pairs for the inverse can be obtained by interchanging the first and second coordinates of the ordered pairs of the original function.

This reinforces the rule that the domain of the inverse function is the range of the original function, and the range of the inverse function is the domain of the original function.



Consider the function defined by the following set of ordered pairs.

$\{(-4,-2), (-2,-1), (-1,0), (0,1), (2,4), (3,8)\}$

a) Describe the inverse of the function by a set of ordered pairs.

$\{(-2,-4), (-1,-2), (0,-1), (1,0), (4,2), (8,3)\}$

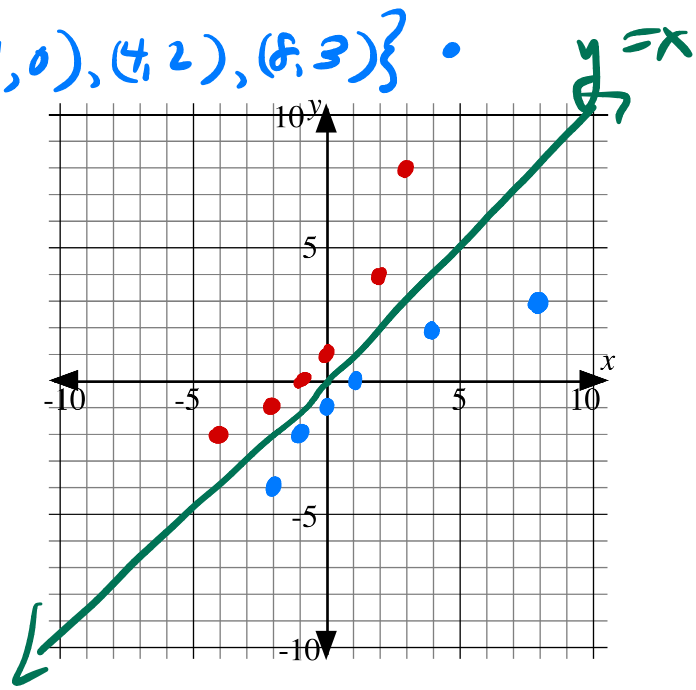
b) Graph the original function and the inverse.

c) Draw a line which acts as a "mirror" between the original function and its inverse.

diagonal reflection

d) State the equation of the mirror line.

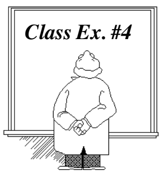
$y = x$



Finding the Inverse of a Function Defined by a Graph

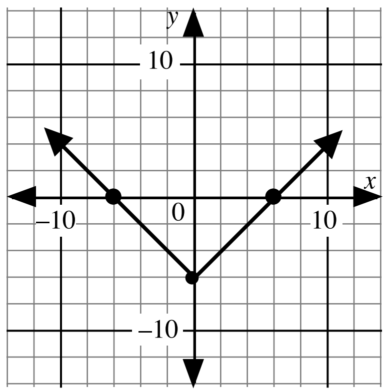
To determine the inverse of a function defined by a graph, reflect the graph of the function in the line $y = x$.

Alternatively, select the coordinates of some key points, interchange the coordinates and plot the new points.

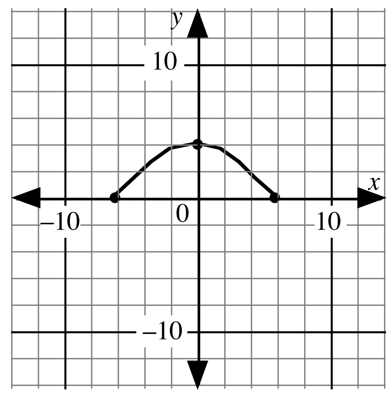


Sketch the graph of the inverse of the functions defined by the following graphs.
Is the inverse a function?

a)



b)

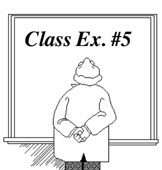


Finding the Inverse of a Function Defined by an Equation - Algebraically

When finding the inverse of a function defined by an equation,

interchange x and y in the equation and then solve for y.

SWAP RE-SOLVE



Consider the function defined by the equation $y = 3x + 2$.

a) Find an equation in the form "$y = mx + \text{b}$" for the inverse of the function.

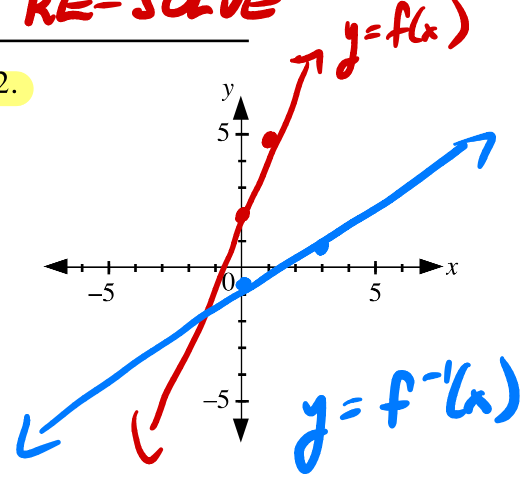
SWAP

$x = 3y + 2$

RE-SOLVE

$y = \frac{x-2}{3}$

or $y = \frac{1}{3}x - \frac{2}{3}$



b) Graph the original and its inverse on the grid.

c) Is the inverse of the function defined by the equation $y = 3x + 2$ also a function?

yes.

$\boxed{\#3\text{-}10}$