Trigonometry - Functions and Graphs Lesson #7:
Transformations of Trigonometric Functions - Part One

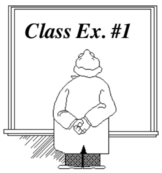
In the next two lessons we will consider the graphs of the functions whose equations are

$$y = a \sin[b(x - c)] + d \quad \text{and} \quad y = a \cos[b(x - c)] + d$$

h K

and relate them to the graphs of the functions whose equations are $y = \sin x$ and $y = \cos x$.

In this lesson we concentrate on the effects of the parameters a and b.



a) Use the knowledge gained from the transformation unit to describe how the graph of the given function compares to the graph of $y = \sin x$, where x is in degrees.

i) $y = 2 \sin x$ vert. stretch by a factor of 2

ii) $y = \sin 2x$ hor. comp. by a factor of $\frac{1}{2}$

iii) $y = -3 \sin x$ reflection on x-axis
vert. exp. by a factor of 3

iv) $y = \sin(-3x)$ refl. on y-axis
hor. comp. by a factor of $\frac{1}{3}$

b) In which of the above examples is there a change in amplitude compared to the graph of $y = \sin x$? i, iii

c) In which of the above examples is there a change in period compared to the graph of $y = \sin x$? ii, iv

d) Complete the table. Use a graphing calculator if necessary.

Equation	Amplitude	Period	
$y = \sin x$	1	2π	$360°$
$y = 2 \sin x$	2	2π	$360°$
$y = \sin 2x$	1	π	$180°$
$y = -3 \sin x$	3	2π	$360°$
$y = \sin(-3x)$	1	$\frac{2\pi}{3}$	$120°$
$y = 5 \sin 4x$	5	$\frac{\pi}{2}$	$90°$
$y = \frac{1}{3} \sin \frac{1}{2}x$	$\frac{1}{3}$	4π	$720°$
$y = a \sin bx$	$\lvert a \rvert$	$\frac{2\pi}{\lvert b \rvert}$	$\frac{360°}{\lvert b \rvert}$

e) Describe the effect of the parameter "a" on the graphs of $y = a \sin x$.

f) Describe the effect of the parameter "b" on the graphs of $y = \sin bx$.

g) Would you expect similar effects on the graph of $y = a \cos bx$? Investigate if necessary. yes.

Effects of a and b in $y = a \sin bx$, $y = a \cos bx$, and $y = a \tan bx$

Changing the parameter "a" on the graphs of $y = a \sin x$ and $y = a \cos x$ results in a vertical stretch about the x-axis with the following:

- If $a > 0$, the result is a vertical stretch of factor a about the x-axis.
- If $a < 0$, the result is a reflection in the x-axis and a vertical stretch of factor $|a|$ about the x-axis.

Changing the parameter "a" on the graph of $y = a \tan x$ also results in a vertical stretch of factor "a" about the x-axis.

Changing the parameter "b" on the graphs of $y = \sin bx$, $y = \cos bx$, and $y = \tan bx$ results in a horizontal stretch about the y-axis with the following:

- If $b > 0$, the result is a horizontal stretch of factor $\frac{1}{b}$ about the y-axis.
- If $b < 0$, the result is a reflection in the y-axis and a horizontal stretch of factor $\frac{1}{|b|}$ about the y-axis.

not necessary. *

$y = a \sin bx$ or $y = a \cos bx$

amplitude $= |a| = \dfrac{\text{Max} - \text{Min}}{2}$

period $= \dfrac{360°}{|b|}$ (for degree measure)

period $= \dfrac{2\pi}{|b|}$ (for radian measure)

$y = a \tan bx$

amplitude is not applicable *

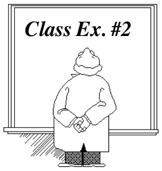
range $y = \mathbb{R}$.

period $= \dfrac{180°}{|b|}$ (for degree measure)

period $= \dfrac{\pi}{|b|}$ (for radian measure)

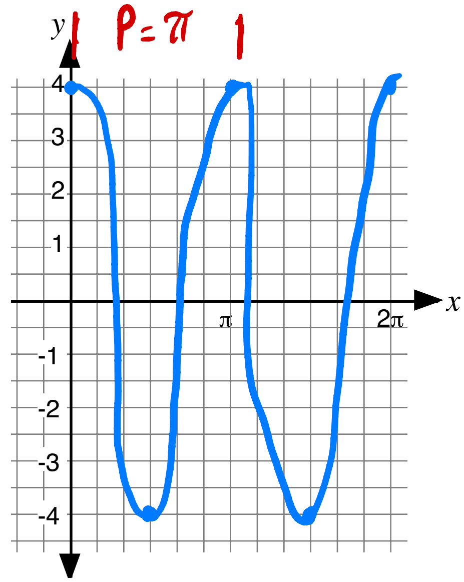
Hints for Graphing a Trigonometric Function Manually

- Sketch the primary trigonometric graph, i.e. $y = \sin x$ or $y = \cos x$.
- Adjust the basic graph for any change in amplitude by considering the max and min points.
- Adjust the new graph for any change in period by dividing the period into four parts using the maximum and minimum points, and the points where the graph intersects the mid-line (the horizontal line running through the centre of the graph).



Class Ex. #2

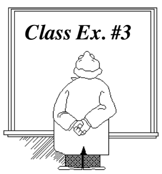
Consider the graph of $y = 4\cos 2x,\ 0 \le x \le 2\pi$.



a) State the amplitude and period.

$|a| = 4$ $\qquad \frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$

b) Sketch the graph on the grid. Use a graphing calculator to verify.



Class Ex. #3

Write the equation of

a) a sine function having an amplitude of $\frac{2}{3}$ and a period of $\frac{\pi}{6}$

$P = \frac{2\pi}{b}$ $\qquad b = \frac{2\pi}{P}$

$a = \frac{2}{3}$ $\quad c = 0$

$b = \frac{2\pi}{\frac{\pi}{6}} = 12$ $\quad d = 0$

$\boxed{y = \frac{2}{3}\sin 12x}$

b) a cosine function having an amplitude of 3 and a period of 720°

$a = 3$ $\quad c = 0$

$b = \frac{360°}{720°} = \frac{1}{2}$ $\quad d = 0$

$\boxed{y = 3\cos \frac{1}{2}x}$

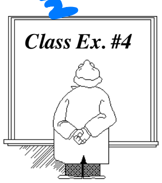
c) a tangent function having a period of $\frac{\pi}{2}$.

$a = 1$ $\quad c = 0$

$b = \frac{\pi}{\frac{\pi}{2}} = 2$ $\quad d = 0$

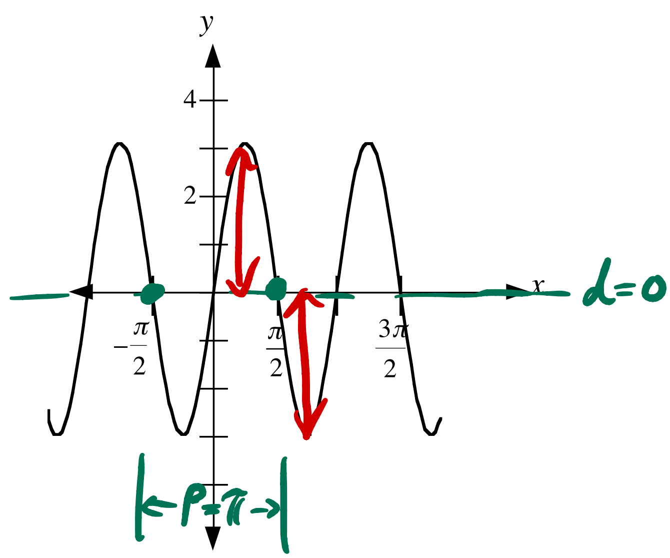
$\boxed{y = \tan 2x}$

* tan function period = π *



Class Ex. #4

Consider the partial graph shown.



a) State the amplitude and period of the graph.

3 $\qquad$ π

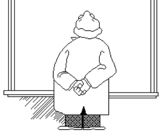
b) Determine the equation of the sine function which the graph represents.

$a = 3$ $\quad c = 0$

$b = \frac{2\pi}{\pi} = 2$ $\quad d = 0$

$\boxed{y = 3\sin 2x}$

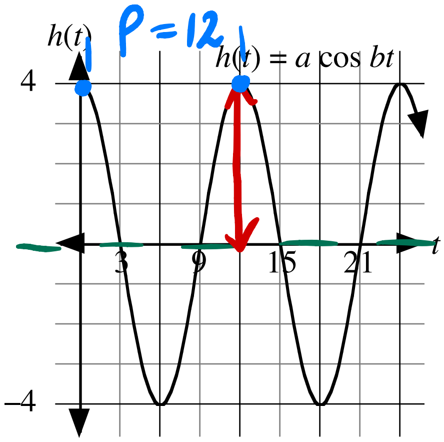
Class Ex. #5



The graph represents the effect of tides on mean sea level over a 24 hour period. The graph has equation $h(t) = a \cos bt$, where t is in hours and h is the height, in metres, relative to mean sea level. Determine the equation of the graph.

$a = 4 \qquad b = \frac{2\pi}{12} = \frac{\pi}{6}$

$\boxed{h(t) = 4 \cos \frac{\pi}{6} t}$



$d=0$

Complete Assignment Questions #1 - #16

#1-13 (a, c, e... where appropriate)

Assignment

1. Describe how the graph of the given function compares to the graph of $y = \cos x$.

a) $y = 5 \cos x$

b) $y = 2 \cos \frac{1}{2}x$

c) $y = -\frac{1}{3} \cos 4x$

d) $y = 0.2 \cos (-6x)$

2. State the amplitude.

a) $y = 5 \sin x$ **b)** $y = \cos 3x$ **c)** $y = \frac{7}{3} \sin 2x$ **d)** $y = -4 \cos \frac{5}{6}\theta$

3. State the period in degrees.

a) $y = 6 \sin x$ **b)** $y = \tan 3x$ **c)** $y = \frac{2}{3} \cos \frac{x}{7}$ **d)** $y = -2 \tan \frac{2}{3}\theta$

4. State the period in radians.

a) $y = 7 \tan x$ **b)** $y = \cos 3x$ **c)** $y = \frac{1}{4} \sin \frac{x}{3}$ **d)** $y = 5 \tan \frac{1}{2}\theta$