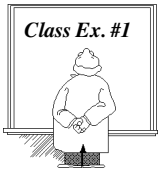
Trigonometry - Functions and Graphs Lesson #8:
Transformations of Trigonometric Functions - Part Two

In this lesson we will consider the graphs of the functions whose equations are

$y = a \sin[b(x - c)] + d$ and $y = a \cos[b(x - c)] + d$

and relate them to the graphs of the functions whose equations are $y = \sin x$ and $y = \cos x$.

In the first part of the lesson we concentrate on the effects of the parameters c and d.



a) Describe how the graph of the given function compares to the graph of $y = \sin x$, where x is in degrees.

i) $y = \sin(x - 30°)$ hor. trans. 30° right

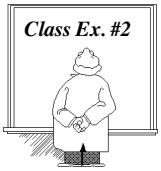
ii) $y = \sin x + 2$ vert. trans 2 units up.

iii) $y = \sin(x + 60°) - 1$ hor. trans. 60° left
vert. trans. 1 unit down

iv) $y - 45 = \sin(x - 45°)$ hor trans. 45° right
vert trans 45 units up.

In trigonometry

- a horizontal translation is called a **horizontal phase shift**, and,
- a vertical translation is called a **vertical displacement**.



Complete the table to describe how the graph of the given function compares to the graph of $y = \sin x$ where x is in radians. Use a graphing calculator if necessary.

c d

Equation	Horizontal Phase Shift	Vertical Displacement
$y = \sin x$	0	0
$y = \sin\left(x - \frac{\pi}{4}\right)$	$\frac{\pi}{4}$ radians right	0
$y = \sin x + 5$	0	5 units up.
$y + \pi = \sin\left(x + \frac{3\pi}{2}\right)$	$\frac{3\pi}{2}$ rads left	π units down.
$y = \sin(x - c) + d$	c units left/right	d units up/down
$y = a \sin[b(x - c)] + d$	" "	" "

Would you expect similar effects on the graph of $y = a \cos[b(x - c)] + d$?
Investigate if necessary.

Yes.

Effects of c and d in $y = a \sin[b(x - c)] + d$ and $y = a \cos[b(x - c)] + d$

Changing the parameter "*c*" on the graphs of $y = a \sin[b(x - c)] + d$ and $y = a \cos[b(x - c)] + d$ results in a horizontal phase shift with the following:

- a horizontal phase shift to the right if $c > 0$
- a horizontal phase shift to the left if $c < 0$

Changing the parameter "*d*" on the graphs of $y = a \sin[b(x - c)] + d$ and $y = a \cos[b(x - c)] + d$ results in a vertical displacement with the following:

- a vertical displacement up if $d > 0$
- a vertical displacement down if $d < 0$

The vertical displacement is determined from a graph using the formula $d = \dfrac{\text{Max} + \text{Min}}{2}$.

Summary of the Effects of the Parameters a, b, c, and d

For $y = a \sin[b(x - c)] + d$
$y = a \cos[b(x - c)] + d$

amplitude $= |a| = \dfrac{\text{Max} - \text{Min}}{2}$

period $= \dfrac{360°}{|b|}$ (for degree measure)

period $= \dfrac{2\pi}{|b|}$ (for radian measure)

horizontal phase shift $= c$
- to the right if $c > 0$
- to the left if $c < 0$

vertical displacement $= d$
- up if $d > 0$
- down if $d < 0$
- $d = \dfrac{\text{Max} + \text{Min}}{2}$

For $y = a \tan[b(x - c)] + d$

amplitude - not applicable

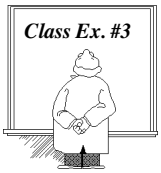
a value represents a vertical stretch of factor $|a|$

period $= \dfrac{180°}{|b|}$ (for degree measure)

period $= \dfrac{\pi}{|b|}$ (for radian measure)

horizontal phase shift $= c$
- to the right if $c > 0$
- to the left if $c < 0$

vertical displacement $= d$
- up if $d > 0$
- down if $d < 0$



Consider equations of the form $y = a\sin[b(x - c)] + d$ and $y = a\cos[b(x - c)] + d$, where $a = 1$, and $b = 1$. Write the equation which represents

a) a cosine function having a horizontal phase shift of 75° right

$x \to x - 75°$

$y = \cos(x - 75°)$

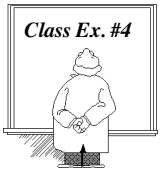
b) a sine function having a horizontal phase shift of $\frac{3\pi}{5}$ radians left, and a vertical displacement 4 units up

$x \to x + \frac{3\pi}{5}$

$y \to y - 4$

$y - 4 = \sin\left(x + \frac{3\pi}{5}\right)$

$y = \sin\left(x + \frac{3\pi}{5}\right) + 4$



Find the amplitude, period, horizontal phase shift, and vertical displacement of the graphs of the following functions defined on $x \in R$.

a) $y = 2\sin 3(x + \pi) - 4$

b) $y = -\frac{2}{3}\cos\frac{1}{4}\left(x - \frac{\pi}{12}\right) + 3$

$a = 2 \to$ amplitude $= 2$

$b = 3 \to P = \frac{2\pi}{3}$

$c = -\pi \to$ phase shift $\to \pi$ radians left

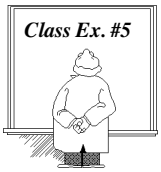
$d = -4 \to$ displacement $\to$ 4 units down

amp. $= \frac{2}{3}$

Period $= 2\pi/\frac{1}{4} = 8\pi$

phase shift $= \frac{\pi}{12}$ rads right

displacement $= 3$ units up.



Find the amplitude, period, horizontal phase shift, and vertical displacement of the graphs of the following functions defined on $x \in R$.

a) $y = 2\sin(3x + \pi) - 4$

$y = 2\sin 3\left(x + \frac{\pi}{3}\right) - 4$

amp $= 2$

per $= 2\pi/3$

ph. shift $= \frac{\pi}{3}$ rads left

disp $=$ down 4 units

b) $y = -\cos\left(2x - \frac{\pi}{2}\right) + \pi$

$y = -\cos 2\left(x - \frac{\pi}{4}\right) + \pi$

amp $= 1$

per $= \pi$

ph shift $= \frac{\pi}{4}$ to right

disp $= \pi$ units up.

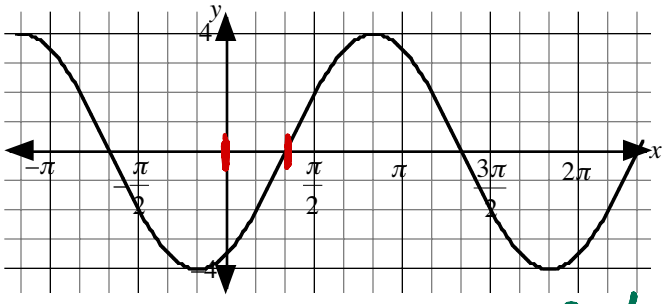
c) Compare the answer to Class Ex. #4a and Class Ex. #5a.

Complete Assignment Questions #1 - #2

Class Ex. #6

The graphs from a) - d) represent the same trigonometric function.

a) Write the equation of the graph in the form $y = a \sin(x - c)$ if $a > 0$ and there is a minimum possible horizontal phase shift.



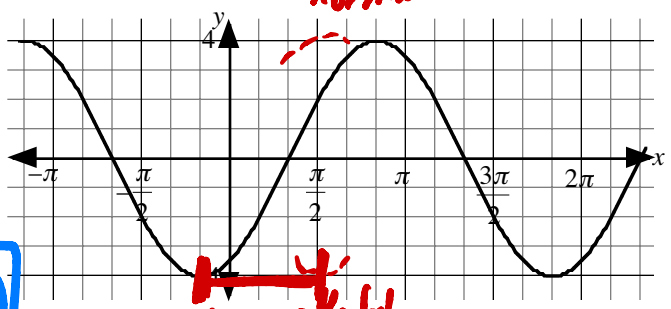
Scale 1 box = $\frac{\pi}{6}$

$a = 4$
$b = \frac{2\pi}{2\pi} = 1$
$c = \frac{\pi}{3}$
$d = 0$

$\frac{2\pi}{6} = \frac{\pi}{3}$ to the right

$y = 4\sin\left(x - \frac{\pi}{3}\right)$

b) Write the equation of the graph in the form $y = a \sin(x - c)$ if $a < 0$ and there is a minimum possible horizontal phase shift.



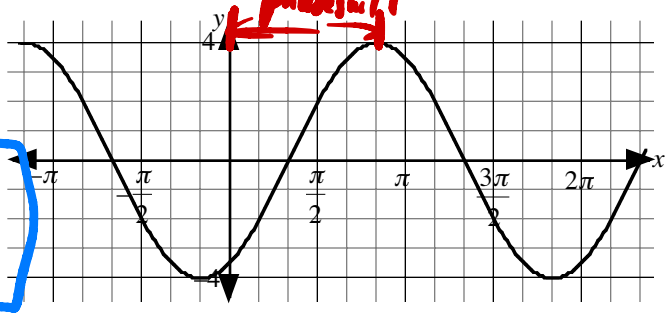
phase shift for reflected sine.

$\frac{4\pi}{6} = \frac{2\pi}{3}$ to the left

$a = -4$
$b = 1$
$c = -\frac{2\pi}{3}$
$d = 0$

$y = -4\sin\left(x + \frac{2\pi}{3}\right)$

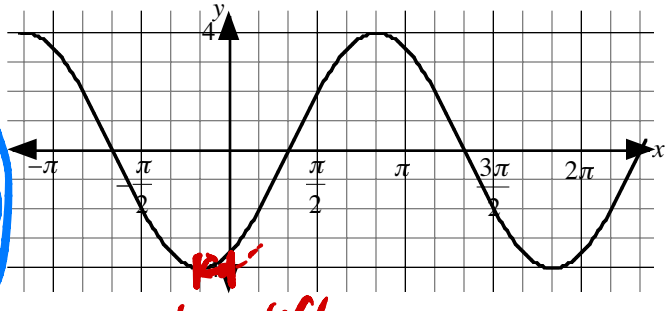
c) Write the equation of the graph in the form $y = a \cos(x - c)$ if $a > 0$ and there is a minimum possible horizontal phase shift.



$a = 4$
$b = 1$
$c = \frac{5\pi}{6}$
$d = 0$

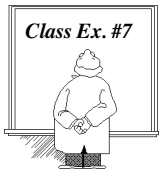
$y = 4\cos\left(x - \frac{5\pi}{6}\right)$

d) Write the equation of the graph in the form $y = a \cos(x - c)$ if $a < 0$ and there is a minimum possible horizontal phase shift.



$a = -4$
$b = 1$
$c = -\frac{\pi}{6}$
$d = 0$

$y = -4\cos\left(x + \frac{\pi}{6}\right)$



Consider the graph shown.

a) If the graph represents a sine function where $a > 0$, complete the table and write the equation represented by the graph.

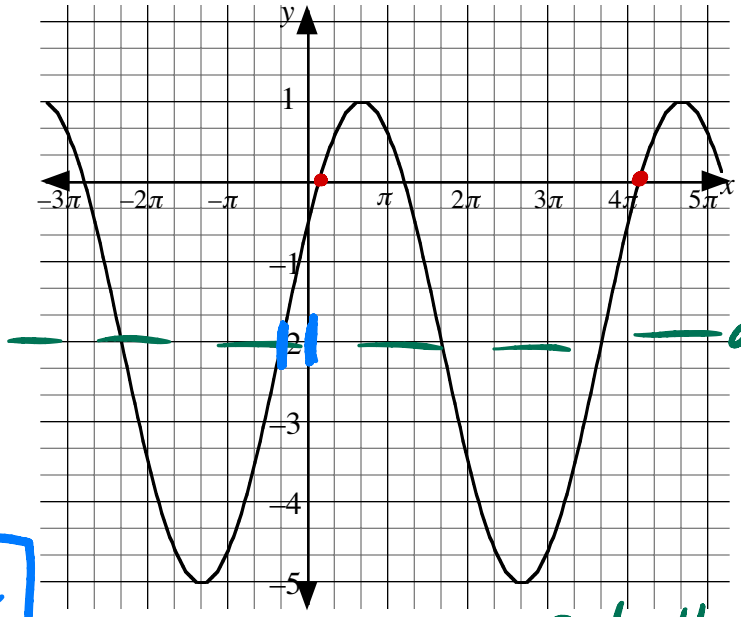
Amplitude	3
Period	4π
Min. Horizontal Phase Shift	$\frac{\pi}{3}$ units left
Vertical Displacement	down 2 units

$a = 3$

$b = \frac{2\pi}{4\pi} = \frac{1}{2}$

$c = -\frac{\pi}{3}$

$d = -2$



$d = -2$

Scale 1 box $= \frac{\pi}{3}$

$$y = 3\sin\tfrac{1}{2}\left(x + \tfrac{\pi}{3}\right) - 2$$

b) Write the equation if the graph in a) represents a sine function where $a < 0$.

c) If the graph represents a cosine function where $a > 0$, complete the table and write the equation represented by the graph.

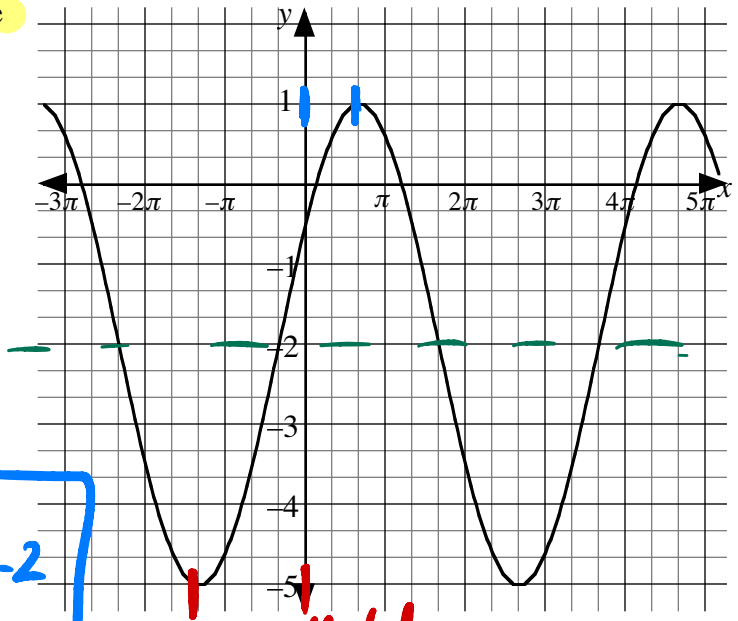
Amplitude	3
Period	4π
Min. Horizontal Phase Shift	$\frac{2\pi}{3}$ right
Vertical Displacement	down 2 units

$a = 3$

$b = \frac{2\pi}{4\pi} = \frac{1}{2}$

$c = \frac{2\pi}{3}$

$d = -2$



$-d = -2$

for reflected cosine $c = -\frac{4\pi}{3}$

$$y = 3\cos\tfrac{1}{2}\left(x - \tfrac{2\pi}{3}\right) - 2$$

d) Write the equation if the graph in c) represents a cosine function where $a < 0$.

$$y = -3\cos\tfrac{1}{2}\left(x + \tfrac{4\pi}{3}\right) - 2$$

Consider the graphs of the functions $y = a \sin [b(x - c)] + d$ and $y = a \cos [b(x - c)] + d$.

a) Changing which of the parameters a, b, c and d affect the

i) domain? none **ii)** range? a, d **iii)** amplitude? a **iv)** period? b **v)** zeros? b, c, d (a if $d \neq 0$)

b) State the maximum and minimum values of the functions in terms of a, b, c, and d, if $a > 0$.

max: $a + d$
min: $-a + d$

c) Determine the range of the function $y = 3 \sin 2(x - \pi) - 4$. $a = 3$ $d = -4$

$-1 \leq y \leq 1$ (default) $\rightarrow$ $-3 \leq y \leq 3$ (amplitude) $\rightarrow$ $-7 \leq y \leq -1$ (displ.)

Complete Assignment Questions #3 - #12

Assignment

#1-12

1. Determine the amplitude, period, horizontal phase shift, and the vertical displacement for each function.

a) $y = \cos\left(x - \frac{\pi}{4}\right) + 3$ **b)** $y = 3 \cos \frac{1}{2}\left(x - \frac{\pi}{2}\right)$ **c)** $y = 3 \cos \frac{1}{2}x - \frac{\pi}{2}$

d) $y = \sin\left(4x - \frac{\pi}{2}\right)$ **e)** $y = -2 \cos 3(x - 45°) + 4$ **f)** $y = 7 \sin\left(\frac{1}{4}x + 20°\right) - 1$