Trigonometry - Functions and Graphs Lesson #9:
Sinusoidal Functions

Sinusoidal Functions

A function whose graph resembles the sine or cosine curve is called a **sinusoidal function**. The graph of a sinusoidal function is called a sinusoidal graph. Many periodic phenomena have sinusoidal graphs, e.g. the time of sunrise as a function of the day of the year, the height of a chair of a ferris wheel as a function of time, the depth of the ocean due to changing tides as a function of time, etc.

In this lesson the equation of the sinusoidal function will be given. In the next lesson we will derive the equation of the sinusoidal function from given information.

Most of the equations used will be functions of time and the variable used will be t. The period of the graph will be in time units. Graphical methods will be used to solve problems and determining a suitable window is an essential feature of the solution.

The depth, d metres, of water in a harbour, t hours after midnight, can be approximated by the function $d(t) = 12 + 5\cos 0.5t$, where $0 \le t \le 24$.

$d(t) = 5\cos 0.5t + 12$

a) Determine the maximum and minimum depths of water in the harbour.

max $(1)\cdot 5 + 12 = 17$m

min $(-1)\cdot 5 + 12 = 7$m

b) Determine the period of the function.

$P = \frac{2\pi}{0.5} = 4\pi$ hrs.

c) Write a suitable window which can be used to display the graph of the function.

x: [0, 24, 2] y: [0, 20, 5]

d) What is the depth of water, to the nearest tenth of a metre at 2:00 a.m.?

let $t = 2$

$d(2) = 5\cos[0.5(2)] + 12$

$= 14.7$m

e) A ship which requires a minimum of 8.5 metres of water is in harbour at midnight. By what time, to the nearest minute, must it leave to prevent grounding?

graphing calc. intersect

4.6923 → 4:41 am.

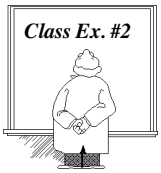
×60

= 7:53

f) What is the next time, to the nearest necessary minute, that the ship can return to the harbour?

$y_1 = 5\cos 0.5t + 12$

$y_2 = 8.5$

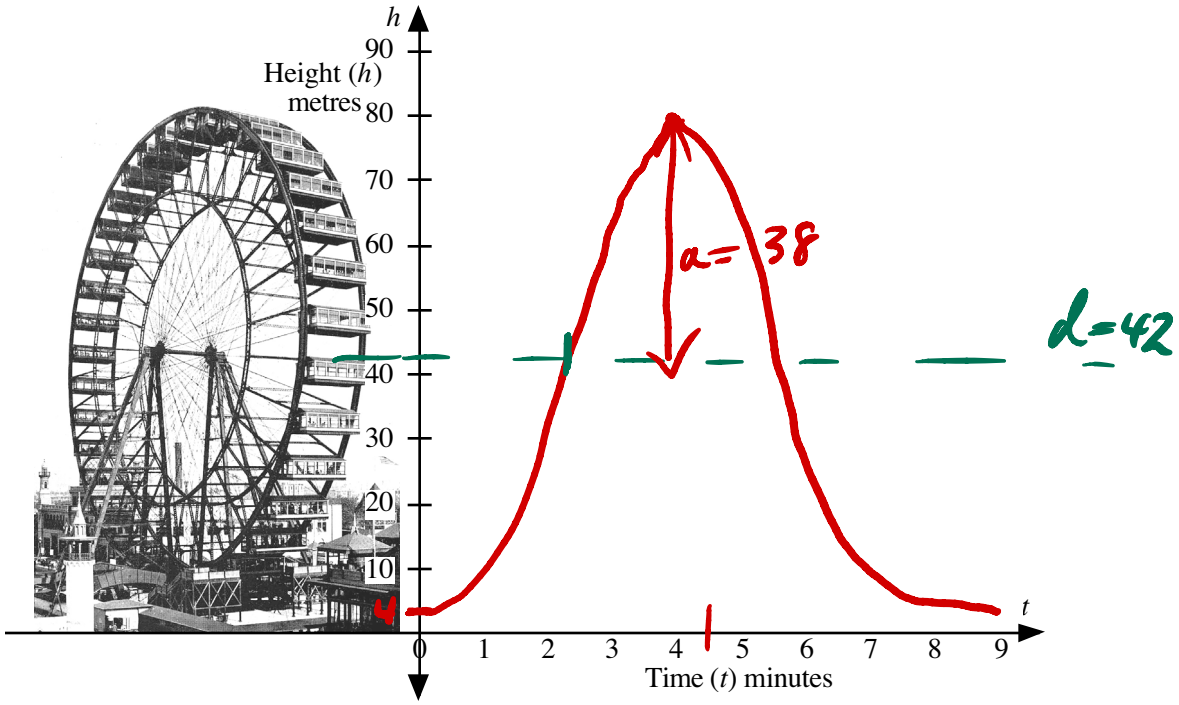


The first Ferris Wheel ever built was created by a bridge builder by the name of George W. Ferris in 1893. The diameter of the wheel was approximately 76 metres and the maximum height of the Ferris Wheel was approximately 80 metres. The wheel had 36 wooden carts on the wheel, with each cart able to hold approximately 60 people

The Ferris Wheel was introduced to the world at the 1893 World's Fair in Chicago. The illustration shown below is a copy of a photograph of the original wheel.

Time (t) minutes	Height (h) metres
0	4
2.25	42
4.5	80
6.75	42
9	4

a) If the wheel rotated every nine minutes, use the data in the table to sketch a sinusoidal graph which represents the height of a car in metres, as a function of time in minutes. Assume that the car is at its lowest point at $t = 0$, and draw one complete cycle.



b) Determine the equation of the graph in the form $h(t) = a \cos b(t - c) + d$.

$a = 38$ $\quad b = \frac{2\pi}{9}$ $\quad c = 4.5$ $\quad d = 42$

$* h(t) = 38 \cos \frac{2\pi}{9}(t - 4.5) + 42$

$* h(t) = -38 \cos \frac{2\pi}{9} t + 42$

$* h(t) = 38 \sin \frac{2\pi}{9}(t - 2.25) + 42$

c) How high, to the nearest metre, is the cart 5 minutes after the wheel starts rotating?

let $t = 5$ algebraically (substitute) value = 5

let y_1 = equation

78m

d) How many seconds after the wheel starts rotating does the cart first reach 10 metres from the ground? Answer to the nearest second.

let y_1 = function. $\quad y_2 = 10$

$x = 0.8159\ldots \times 60$

$= 49$ seconds

Complete Assignment Questions #1 - #5

#1-2 & Problem Sets due end of class Thursday.