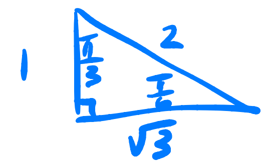
Trigonometry - Equations and Identities Lesson #1:
Solving First Degree Trigonometric Equations

Overview

In this unit, we will

- solve, algebraically and graphically, first and second degree trigonometric equations expressed in degrees and radians, with
  - i) a restricted domain
  - ii) an unrestricted domain leading to a general solution
- prove trigonometric identities using reciprocal identities, quotient identities, Pythagorean identities, sum or difference identities, and double angle identities.



Review

Use an algebraic procedure to solve the following equations on the given domain.

a) $\sin x = -\frac{1}{2}$, $0 \le x \le 2\pi$. exact

III IV

$\text{ref}\angle = \frac{\pi}{6}$

III: $x = \frac{7\pi}{6}$ IV: $x = \frac{11\pi}{6}$

b) $3\sec x - 5 = 0$, $0° \le x \le 360°$, to the nearest degree

$\frac{3\sec x}{3} = \frac{5}{3}$

$\sec x = 5/3$

$\cos x = 3/5$ $\cos^{-1}\left(\frac{3}{5}\right) = 53°$

I IV

$x = 53°$ $x = 307°$

General Solution

The **general solution** to a trigonometric equation is the solution over the **domain of real numbers**. We will investigate how to determine a general solution graphically and algebraically in this lesson.

General Solution Using an Algebraic Approach

Use the following procedure to find the general solution using an algebraic approach.

1. Solve the equation where the domain is **one period** of the graph of the function.
2. The general solution can be determined by adding or subtracting **multiples of the period** to the solutions in 1. ie. coterminals

Class Ex. #2

Use an algebraic procedure to find the general solution of the equation $2\cos x - \sqrt{3} = 0,\ x \in R$, where x is in radian measure. exact

$\frac{2\cos x}{2} = \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$

ref $\angle = \frac{\pi}{6}$ in I, IV

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

$x = \frac{\pi}{6} + 2\pi n,\ \frac{11\pi}{6} + 2\pi n \quad n \in I$

Class Ex. #3

In some cases, the different parts of a general solution can be combined together in one. Determine the general solution, in radians, of the equation

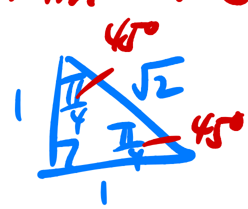
a) $\sin x = 0$ **b)** $\cos x = 0$

a) $x = 0, \pi, 2\pi$

$x = \pi n \quad n \in I$

b) $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{2} + \pi n \quad n \in I$



Solving on a Restricted Domain

Use the following procedure to solve a trigonometric equation on a restricted domain.

1. Determine the period of the trigonometric function.
2. Solve the equation on the domain $0 \le x \le$ period.
3. Add or subtract **multiples of the period** to the solutions in 1 to solve in **the restricted domain**.

Class Ex. #4

Solve the following equations on the specified domain.

a) $2\sin x - \sqrt{2} = 0$ for $360° \le x \le 720°$ exact

b) $\sqrt{3}\cot x + 1 = 0$ for $-\pi \le x \le 0$ exact

a) $\frac{2\sin x}{2} = \frac{\sqrt{2}}{2}$

$\sin x = \frac{\sqrt{2}}{2}$

ref $\angle = 45°$ I & II

$x = 45°, 135°$
$+360° \quad +360°$

$x = 405°, 495°$

b) $\frac{\sqrt{3}\cot x}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$\cot x = \frac{-1}{\sqrt{3}}$

$\tan x = -\sqrt{3}$

ref $\angle = \frac{\pi}{3}$ II, IV

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$
$-\frac{6\pi}{3} \quad -\frac{6\pi}{3}$

$x = \cancel{-\frac{4\pi}{3}}, \boxed{-\frac{\pi}{3}}$

Complete Assignment Questions #1 - #15

$\boxed{\#7-11}$