Trigonometry - Equations and Identities Lesson #2:
Solving Second Degree Trigonometric Equations

In this lesson we will be solving **second degree** equations where the power of the trigonometric function is two (e.g. $\sin^2 x - 3\sin x = 0$).

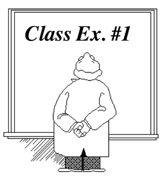
Trigonometric equations which can be solved by using identities will be covered in lesson 6.

Factoring Trigonometric Expressions

Just as with polynomial expressions, trigonometric expressions can be factored.
The ability to factor trigonometric expressions is a useful skill in two areas:

- solving trigonometric equations (in this lesson)
- proving trigonometric identities (in lesson 6)

In factoring trigonometric expressions we can apply three basic factoring techniques - common factor, difference of two squares, and trinomials of the form $ax^2 + bx + c$, $a \neq 0$.



Factor the following trigonometric expressions:

a) $8\tan A + 4$

$4(2\tan A + 1)$

b) $\sin^2 x - 3\sin x$

$\sin x(\sin x - 3)$

c) $4\sin^2 x - 1$

$(2\sin x + 1)(2\sin x - 1)$

d) $\csc^2 x - 3\csc x - 28$

let $n = \csc x$

$n^2 - 3n - 28$

$(n - 7)(n + 4)$

$(\csc x - 7)(\csc x + 4)$

e) $2\cos^2 x + 7\cos x - 4$

let $n = \cos x$

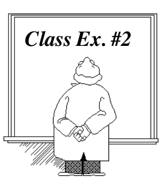
$2n^2 + 7n - 4$

$(2n - 1)(n + 4)$

$(2\cos x - 1)(\cos x + 4)$

Complete Assignment Question #1

Solving a Second Degree Equation Using a Graphical Approach

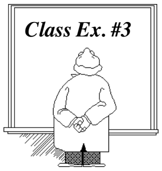


Consider the equation $2\sin^2 x = 1 - \sin x$.

a) Use a **graphical** approach to determine the roots of the equation where $0 \leq x \leq 2\pi$.
Give solutions as exact multiples of π.

b) State the general solution to the equation.

Solving a Second Degree Equation Using an Algebraic Approach



set to zero

Consider the equation $2\sin^2 x = 1 - \sin x$.

a) Use an **algebraic** approach to determine the roots of the equation where $0 \le x \le 2\pi$. Give solutions as exact values.

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

zero product law

$2\sin x - 1 = 0 \qquad \sin x = -1$

$\sin x = \frac{1}{2}$

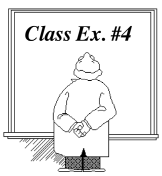
ref∠ $= \frac{\pi}{6}$

I & II

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

b) State the general solution to the equation.

$x = \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, \frac{3\pi}{2} + 2\pi n \qquad n \in I$



Use an algebraic procedure to determine the roots of the equation on the given domain and write the general solution.

a) $4\sin^2 A - 1 = 0, 0 \le A \le 2\pi$

$(2\sin A + 1)(2\sin A - 1) = 0$

$\sin A = -\frac{1}{2} \qquad \sin A = \frac{1}{2}$

ref∠ $= \frac{\pi}{6}$ in III, IV $\qquad$ ref∠ $= \frac{\pi}{6}$ in I, II

$A = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

gen. sol: $A = \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n,$
$\frac{7\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$

b) $\tan^2 x + \tan x = 0, \quad 0 \le x \le 2\pi$

$\tan x(\tan x + 1) = 0$

$\tan x = 0 \qquad \tan x = -1$

$x = 0, \pi, 2\pi$

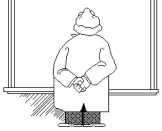
ref∠ $= \frac{\pi}{4}$

II, IV

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = \pi n, \frac{3\pi}{4} + \pi n$

Class Ex. #5



Determine the zeros of the following functions on the specified domain.

a) $f(x) = \csc^2 x - 3\csc x - 28$, domain $0° \le x \le 180°$ I & II only
Answer to the nearest degree

$0 = \csc^2 x - 3\csc x - 28$

$0 = (\csc x - 7)(\csc x + 4)$

$\csc x = 7 \qquad \csc x = -4$

$\sin x = \frac{1}{7} \qquad \sin x = -\frac{1}{4}$ X

$\sin^{-1}\left(\frac{1}{7}\right) = \text{ref} \angle = 8°$

I & II

$\boxed{x = 8°, 172°}$

b) $g(\theta) = 2\cos^2\theta + 5\cos\theta - 3$, domain $-\pi \le \theta \le \pi$

Complete Assignment Questions #2 - #13

1-3, 7, 8

Assignment

1. Factor the following trigonometric expressions.

a) $4\sin^2\theta - \cos^2\theta$ **b)** $\cot^2 x - \cot x$ **c)** $\sin^2\theta + 3\sin\theta + 2$

d) $\sec x \sin^2 x - 0.25\sec x$ **e)** $\cot^2\theta - 1$ **f)** $\sec^4\theta - 1$

g) $4\cos^2 A - 4\cos A - 3$ **h)** $2\sin^2 x - 7\sin x + 6$