

Solving a Multiple Angle Equation Using a Graphical Approach



- a) Given $\tan 2x = \sqrt{3}$, where $0 \leq x \leq 2\pi$, find the exact values of x using a graphical approach.
- b) State the general solution to the equation $\tan 2x = \sqrt{3}$.
- c) Complete the following statement.
 The general solution consists of answers which differ by _____ radians because the graph of $y = \tan 2x$ has a **period** of _____ radians.

Complete Assignment Questions #1 - #3

Algebraically Investigating Solutions to Multiple Angle Equations

Consider the equation $\sin 3x = \frac{\sqrt{2}}{2}$.

$P = \boxed{\frac{2\pi}{3}} = \frac{8\pi}{12}$

- a) Complete the following to solve the equation $\sin 3x = \frac{\sqrt{2}}{2}$, where $0 \leq x \leq 2\pi$.

• If x is defined for domain $0 \leq x \leq 2\pi$, then $3x$ is defined for domain $0 \leq 3x \leq 6\pi$.

$\sin 3x = \frac{\sqrt{2}}{2}$ Quadrants **I** and **II** Reference angle = $\frac{\pi}{4}$

$3x = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ or $2\pi + \underline{\hspace{1cm}}$ or $2\pi + \underline{\hspace{1cm}}$ or $4\pi + \underline{\hspace{1cm}}$ or $4\pi + \underline{\hspace{1cm}}$

$3x = \frac{\pi}{4}$ $3x = \frac{3\pi}{4} + \frac{2\pi}{3}$ $3x = \frac{3\pi}{4} + \frac{2\pi}{3}$

$x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$

- b) State the general solution to the equation $\sin 3x = \frac{\sqrt{2}}{2}$.

$x = \frac{\pi}{12} + \frac{2\pi}{3}n, \frac{\pi}{4} + \frac{2\pi}{3}n \quad n \in \mathbb{I}$

- c) Verify the solution using a graphical approach.



• The general solution consists of two sets of answers which differ by $\frac{2\pi}{3}$ radians because the graph of $y = \sin 3x$ has a **period** of $\frac{2\pi}{3}$ radians.

Solving a Multiple Angle Equation Using an Algebraic Approach

Use the following procedure to solve multiple angle equations.

1. Find the domain for the multiple angle.
2. Solve for the multiple angle between 0 and 2π using the CAST rule and reference angle.
3. Add the period of the trigonometric graph of the multiple angle to each of the answers in step 2 until you cover the domain in step 1.



Class Ex. #2

Consider the equation $\sin 2x = -\frac{1}{2}$.

- a) Find the exact values of x using an algebraic approach where $0 \leq x \leq 2\pi$.

$$\text{ref } \angle = \frac{\pi}{6} \text{ in III \& IV}$$

$$2x = \frac{7\pi}{6} \qquad 2x = \frac{11\pi}{6}$$

$$x = \frac{7\pi}{12} + \frac{12\pi}{12} \qquad x = \frac{11\pi}{12} + \frac{12\pi}{12}$$

$$x = \frac{19\pi}{12} \qquad x = \frac{23\pi}{12}$$

$$x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

$$p = \frac{2\pi}{2} = \pi$$

$$\downarrow$$

$$\frac{12\pi}{12}$$

- b) State the general solution to the equation $\sin 2x = -\frac{1}{2}$.

$$x = \frac{7\pi}{12} + \pi n, \frac{11\pi}{12} + \pi n \quad n \in \mathbb{I}$$

- c) Complete the following statement.

The general solution consists of answers which differ by π radians because the graph of $y = \sin 2x$ has a period of π radians.

- d) Verify the solution using a graphical approach.

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