Trigonometry - Equations and Identities Lesson #4:
Trigonometric Identities - Part One

Equations and Identities

In mathematics it is important to understand the difference between an equation and an identity.

$2x^2 + 3 = 11$ is an **equation**. It is only true for <u>certain values</u> of the variable x.
The solutions to this equation are –2 and 2 which can be verified by substituting these values into the equation.

$(x + 1)^2 = x^2 + 2x + 1$ is an **identity.** It is true for <u>all values</u> of the variable x. for which x is defined

Reviewing Identities

Recall the basic trigonometric identities:

Basic Identities

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \qquad \text{where } x^2 + y^2 = r^2$$

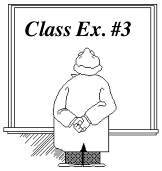
We have also met the reciprocal trigonometric identities :

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

We can use the Basic and Reciprocal trigonometric identities to prove the Quotient and Pythagorean identities.

Before doing this we will verify some identities using a particular case.



Use the basic identities to prove the identity $1 + \tan^2 A = \sec^2 A$.

In the same way the basic identities can be used to prove the following:

Quotient Identities

$\tan x = \dfrac{\sin x}{\cos x}$ $\qquad$ $\cot x = \dfrac{\cos x}{\sin x}$

* all identities needed are on your formula sheet *

unit circle
$r=1$
$\sin\theta = y$
$\cos\theta = x$
$\tan\theta = \dfrac{y}{x}$

and

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $1 + \tan^2 x = \sec^2 x$ $\qquad$ $1 + \cot^2 x = \csc^2 x$

- These identities can be written in several ways and this should be remembered in trying to prove more difficult identities in the next lesson. For example

$\sin^2 x = 1 - \cos^2 x \qquad \cos^2 x = 1 - \sin^2 x$

$\tan^2 x = \sec^2 x - 1 \qquad \cot^2 x = \csc^2 x - 1 \qquad$ etc.

- We use the basic trigonometric identities in terms of x, y and r to prove **only** the Quotient and Pythagorean Identities.
- You will be asked to verify the remaining Quotient and Pythagorean Identities in the Assignment.
- Before considering more complex identities in the next lesson we need to review some skills in simplification and factoring which will help in the proofs.

Complete Assignment Questions #1 - #5

Using Identities to Simplify Trigonometric Expressions

* change to sin & cos
* common denominator
* conjugate multiplication

Class Ex. #4

Express each as a single trigonometric ratio. Use a graphing calculator to verify.

a) $\dfrac{\sin^2 x}{\cos^2 x} + 1$ $= \dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \boxed{\sec^2 x}$

b) $\sin x + \cot x \cos x$ $= \sin x + \dfrac{\cos x}{\sin x} \cdot \cos x$

$= \dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x} = \dfrac{1}{\sin x} = \boxed{\csc x}$

Class Ex. #5

Express $\dfrac{2 \tan A}{1 + \tan^2 A}$ in terms of sin A and cos A and write in simplest form.

$\dfrac{2\tan A}{\sec^2 A} = \dfrac{\frac{2\sin A}{\cos A}}{\frac{1}{\cos^2 A}}$ $\quad \dfrac{2\sin A}{\cancel{\cos A}} \cdot \dfrac{\cos^{\cancel{2}} A}{1}$

$= 2\sin A\cos A$

Class Ex. #6

Factor the following trigonometric expressions.

a) $3\cos^4\theta - 3\sin^4\theta$

$3(\cos^4\theta - \sin^4\theta)$

$3(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

1

$\boxed{3(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$

b) $\sin^2\theta + \sin^2\theta \cot^2\theta$

$\sin^2\theta(1 + \cot^2\theta)$

$\sin^2\theta \csc^2\theta$

$\cancel{\sin^2\theta}\left(\dfrac{1}{\cancel{\sin^2\theta}}\right)$

$= \boxed{1}$

Complete Assignment Questions #6 - #17

Assignment

#6, 8-15

1. Verify the following identities for the given value of the variable.

a) $\cot x = \dfrac{\cos x}{\sin x}$ for $x = 60°$

b) $\sin^2 x + \cos^2 x = 1$ for $x = \dfrac{\pi}{4}$