Non-Permissible Values of an Identity

* never divide by zero *

The non-permissible values of an identity are determined by finding the non-permissible values for each side of the identity.

When determining the non-permissible values, there are two areas to focus on.

1. All tangent, cotangent, secant, and cosecant functions have non-permissible values.

   - $\tan x = \dfrac{\sin x}{\cos x}$ and $\sec x = \dfrac{1}{\cos x}$ have non-permissible values when $\cos x = 0$. $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$
   - $\cot x = \dfrac{\cos x}{\sin x}$ and $\csc x = \dfrac{1}{\sin x}$ have non-permissible values when $\sin x = 0$. $x \neq 0, \pi, 2\pi$

2. If the identity has a denominator, then any zero of the denominator will be a non-permissible value of the identity.

- The **non-permissible values** are sometimes called the **restrictions** of the identity.
- In many of the examples, the restrictions are the solutions to $\sin x = 0$ and/or $\cos x = 0$.
- Recall $\sin x = 0 \Rightarrow x = n\pi, n \in I$, and $\cos x = 0 \Rightarrow x = \dfrac{\pi}{2} + n\pi, n \in I$, (see page 594).

on x-axis on y-axis

In this lesson we will verify and prove more complex trigonometric identities using the skills we learned from the previous lesson.
Some useful steps or hints when trying to prove a trigonometric identity are listed below.

Hints in Proving an Identity

Δ to sine & cosine

1. Begin with the more complex side. or work both sides
2. If possible, use the reciprocal, quotient, or Pythagorean identities. For example, use the Pythagorean identities when squares of trigonometric functions are involved.
3. If necessary change all trigonometric ratios to sines and/or cosines.
   For example, replace $\tan x$ by $\dfrac{\sin x}{\cos x}$, and $\sec x$ by $\dfrac{1}{\cos x}$.
4. Look for factoring as a step in trying to prove an identity.
5. If there is a sum or difference of fractions, combine as a single fraction. common denominators

* multiply by the conjugate *
ie. $(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)$

Consider the statement $\frac{1}{\cos x} - \cos x = \sin x \tan x$.

a) Verify the statement is true for $x = \frac{\pi}{3}$.

L.S.	R.S.
$\frac{1}{\frac{1}{2}} - \frac{1}{2}$	$\frac{\sqrt{3}}{2}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$
$2 - \frac{1}{2}$	$\frac{\sqrt{3}}{2} \cdot \left(\frac{\sqrt{3}}{2} \div \frac{1}{2}\right)$
$\frac{3}{2}$	$\frac{\sqrt{3}}{2} \cdot \sqrt{3}$
	$\frac{3}{2}$

$3/2 = 3/2$ LS=RS.

b) Use a graphing calculator to show that the statement is probably an identity.

$y_1 = \frac{1}{\cos x} - \cos x$

$y_2 = \sin x \tan x$

c) Prove the statement is an identity using an algebraic approach.

$\frac{1}{\cos x} - \cos x$ $\qquad$ $\sin x \tan x$

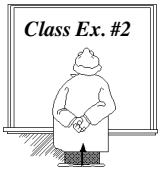
L.S.	R.S.
$\frac{1}{\cos x} - \frac{\cos x}{\cos x}$	$\sin x \cdot \frac{\sin x}{\cos x}$
$\frac{1-\cos^2 x}{\cos x}$	$\frac{\sin^2 x}{\cos x}$
$\frac{\sin^2 x}{\cos x}$	

LS=RS.

d) State the restrictions in terms of x.

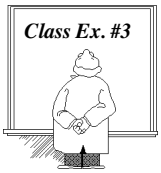
$\cos x \neq 0$

$x \neq \frac{\pi}{2} + \pi n$, $n \in I$



Prove the identity $\sin x \cos^2 x + \sin^3 x = \frac{1}{\csc x}$ algebraically and determine the non-permissible values

L.S.	R.S.



Consider the identity $\dfrac{\sin A + \tan A}{1 + \cos A} = \dfrac{1}{\cot A}$.

a) Prove the identity algebraically.

$\dfrac{\sin A + \tan A}{1+\cos A}$ | $\dfrac{1}{\cot A}$

$\dfrac{\sin A + \frac{\sin A}{\cos A}}{1+\cos A}$ | $\tan A$

$\dfrac{\frac{\sin A \cos A}{\cos A} + \frac{\sin A}{\cos A}}{1+\cos A}$ | $\dfrac{\sin A}{\cos A}$

$\dfrac{\frac{\sin A \cos A + \sin A}{\cos A}}{1+\cos A}$

$\dfrac{\sin A(\cos A + 1)}{\cos A} \cdot \dfrac{1}{1+\cos A}$

$\dfrac{\sin A}{\cos A}$

LS = RS

b) What are the non-permissible values of the identity.

$\sin A \neq 0$ $\quad 1+\cos A \neq 0$ $\quad \cot A \neq 0$ $\quad \cos A \neq 0$

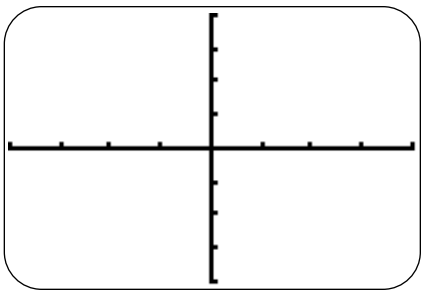
$A \neq \pi n$ $\quad \cos A \neq -1$ $\quad \tan A \neq$ undefined. $\quad A = \frac{\pi}{2} + \pi n$

$A = \pi$

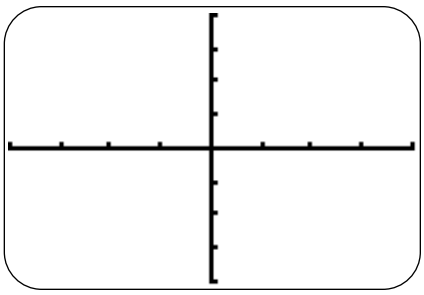
$A \neq \pi n, \ \frac{\pi}{2} + \pi n$

c) Show the non-permissible values on a graph of each side of the identity for the domain $0 \le x \le 2\pi$.

$y = \dfrac{\sin A + \tan A}{1 + \cos A}$



$y = \dfrac{1}{\cot A}$



Prove the identity $\dfrac{\sec^2 x}{\sec^2 x - 1} = \csc^2 x$.

$\dfrac{\sec^2 x}{\tan^2 x}$ | $\dfrac{1}{\sin^2 x}$

$\dfrac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}}$

$\dfrac{1}{\cos^2 x} \cdot \dfrac{\cos^2 x}{\sin^2 x}$

$\dfrac{1}{\sin^2 x}$

LS = RS.

Complete Assignment Questions #1 - #7

#3-4