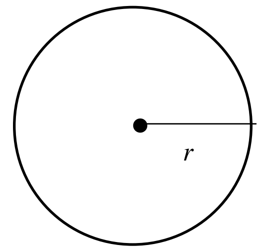
Converting Between Degrees and Radians

Since an angle can be measured in degrees or radians, it is important to be able to convert from one measure to the other.

Consider a circle with a radius of r units. Complete the following:



a) i) One complete rotation in degrees is $\underline{360^\circ}$.

ii) The arc length for one complete rotation is $\underline{2\pi r}$ which is the $\underline{\text{circumference}}$ of the circle.

iii) The ratio, $\dfrac{\text{arc length}}{\text{radius}} = \dfrac{2\pi r}{r} = 2\pi$ **iv)** $360° = \underline{2\pi}$ radians.

b) i) One-half rotation in degrees is $\underline{180^\circ}$.

ii) The arc length for one-half rotation is $\underline{\pi r}$.

iii) The ratio, $\dfrac{\text{arc length}}{\text{radius}} = \dfrac{\pi r}{r} = \pi$ **iv)** $180° = \underline{\pi}$ radians.

c) i) π radians = 180 degrees

so 1 radian = $\dfrac{180}{\pi}$ degrees

To convert from radians to degrees,

multiply by $\dfrac{180^\circ}{\pi}$.

ii) 180 degrees = π radians

so 1 degree = $\dfrac{\pi}{180^\circ}$ radians

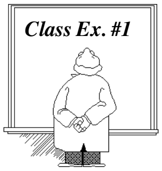
To convert from degrees to radians,

multiply by $\dfrac{\pi}{180^\circ}$.

- In mathematics, the symbol "°" following a number means the unit of angular measure is degrees.
- If there is no unit after the number, or there is the abbreviation "rad", or the word radians, then the unit is radians.
- For example, if you wish to write the sine ratio for a right angle, you must write sin 90°, and NOT sin 90.

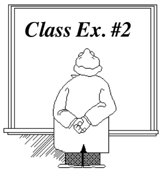
Converting Between Degrees and Radians

- π radians = 180°
- <u>Degrees to Radians</u> multiply by $\dfrac{\pi}{180}$
- <u>Radians to Degrees</u> multiply by $\dfrac{180}{\pi}$



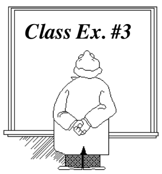
Convert from degrees to radians (give your answer as an exact value in terms of π).

a) $270\cancel{^{\circ}} \times \frac{\pi}{180\cancel{^{\circ}}} = \boxed{\frac{3\pi}{2}}$ **b)** $315\cancel{^{\circ}} \times \frac{\pi}{180\cancel{^{\circ}}} = \boxed{\frac{7\pi}{4}}$



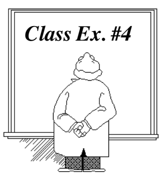
Convert the following from degrees to radians (to the nearest tenth).

a) $70\cancel{^{\circ}} \times \frac{\pi}{180\cancel{^{\circ}}} \approx \boxed{1.2 \text{ rads}}$ **b)** $205\cancel{^{\circ}} \times \frac{\pi}{180\cancel{^{\circ}}} \approx \boxed{3.6 \text{ rads}}$



Convert the following from radians to degrees.

a) $\frac{\cancel{\pi}}{4} \times \frac{180^{\circ}}{\cancel{\pi}} = \boxed{45^{\circ}}$ **b)** $\frac{-7\cancel{\pi}}{\cancel{3}_1} \times \frac{\cancelto{60^{\circ}}{180^{\circ}}}{\cancel{\pi}} = \boxed{-420^{\circ}}$



Convert the following from radians to degrees (to the nearest tenth).

a) 1.57 radians $\times \frac{180^{\circ}}{\pi} \approx \boxed{90.0^{\circ}}$ **b)** −1.4 rad $\times \frac{180^{\circ}}{\pi} \approx \boxed{-80.2^{\circ}}$

Complete Assignment Questions #1 - #4

Coterminal Angles and Reference Angles in Radians

Fill in the blanks in the statements and table below using radian measure.

- **Coterminal angles** are angles with the same terminal arm.
  They are separated by a multiple of 360°, or 2π radians.

- The **principal angle** of a set of coterminal angles is the smallest positive rotation angle with the same terminal arm.
  The principal angle is between 0° and 360°, or between 0 radians and 2π radians.

- A **reference angle** is the acute angle formed between he terminal arm of the rotation angle and the *x*-axis. The relationship between rotation angle and reference angle in each quadrant is given in the table below.

Quadrant	**Relationship in Degrees Rotation Angle =**	**Relationship in Radians Rotation Angle =**
One	Reference Angle	reference angle.
Two	180° – Reference Angle	π − ref ∠
Three	180° + Reference Angle	π + ref ∠
Four	360° – Reference Angle	2π − ref ∠

Class Ex. #5

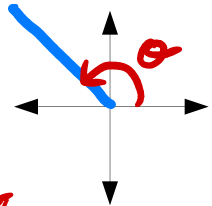
In each of the following

i) draw the angle θ in standard position **ii)** state the principal angle

iii) determine one positive and one negative coterminal angle for the angle θ

iv) write an expression involving the principal angle that represents all angles in the domain $\theta \in R$ that are coterminal with the given angle

a) $\theta = \dfrac{3\pi}{4}$



i.)

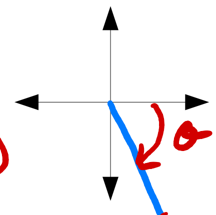
ii.) $\frac{3\pi}{4}$

iii.) $\frac{3\pi}{4} + \frac{8\pi}{4} = \frac{11\pi}{4}$

$\frac{3\pi}{4} - \frac{8\pi}{4} = -\frac{5\pi}{4}$

iv.) $\frac{3\pi}{4} + 2\pi n \quad n \in I$

b) $\theta = -\dfrac{\pi}{3}$



i.)

ii.) $-\frac{\pi}{3} + \frac{6\pi}{3} = \frac{5\pi}{3}$

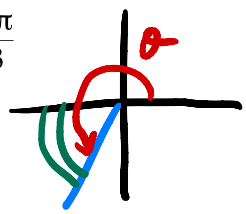
iii.) $\frac{5\pi}{3}$, $-\frac{\pi}{3} - \frac{6\pi}{3} = -\frac{7\pi}{3}$

iv.) $\frac{5\pi}{3} + 2\pi n \quad n \in I$

Class Ex. #6

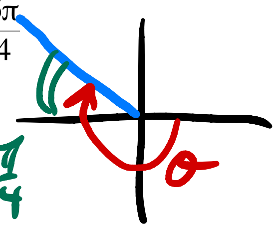
Determine the reference angle for the following rotation angles.

a) $\dfrac{4\pi}{3}$



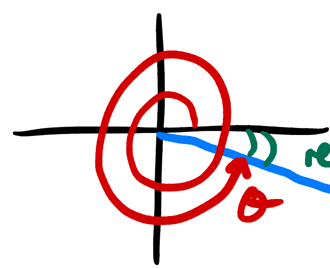
ref L $= \frac{\pi}{3}$

b) $-\dfrac{5\pi}{4}$



ref L $= \frac{\pi}{4}$

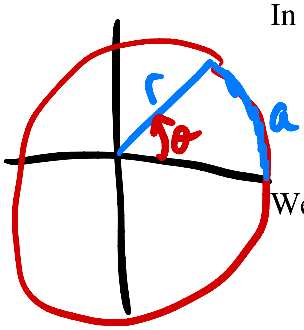
c) $\dfrac{23\pi}{6}$



ref L $= \frac{\pi}{6}$

Complete Assignment Questions #5 - #7

Arc Length



In this lesson we defined the radian measure of an angle, θ, as

$$\text{radian measure} = \frac{\text{length of arc forming the angle}}{\text{length of radius}} = \frac{\text{arc length}}{\text{radius}}, \quad \text{i.e. } \theta = \frac{a}{r}.$$

We can rearrange the formula $\theta = \dfrac{a}{r}$ in terms of arc length as

$\boxed{a = r\theta}$ where θ = the measure of the angle in **radians**,

a = the length of the arc around the angle, and

r = the length of the radius.

* only in radians *

This formula can be used to solve problems involving arc length, radius, and central angle, provided the angle is measured in radians.

Class Ex. #7

A pendulum 30 cm long swings through an arc of 45 cm. Through what angle does the pendulum swing? Answer in degrees and in radians to the nearest tenth.

$a = r\theta$

$45 = 30\theta$

$\frac{45}{30} = \frac{30\theta}{30}$

$\theta = 1.5$ rads

$1.5 \cancel{\text{rads}} \times \frac{180°}{\pi \cancel{\text{rads}}}$

$\doteq \boxed{85.9°}$

Class Ex. #8

Calculate the arc length (to the nearest tenth of a metre) of a sector of a circle with diameter 9.2 m if the sector angle is 150°.

$150\cancel{°} \times \frac{\pi}{180\cancel{°}} = \frac{5\pi}{6}$ rads

$a = r\theta$

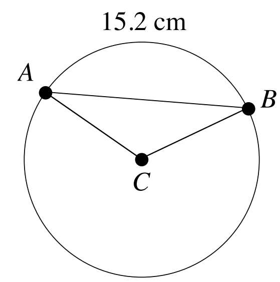
$= 4.6\left(\frac{5\pi}{6}\right)$

$= \boxed{12.0\text{m}}$

Class Ex. #9

A circle with centre *C* and minor arc *AB* measuring 15.2 cm is shown.

If $\angle ABC = \angle BAC = \frac{\pi}{6}$ radians, find the length of the radius of the circle to the nearest tenth of a centimetre.



Complete Assignment Questions #8 - #17

$\boxed{\#2\text{–}15}$

Assignment

1. The diagram shows a series of rotation angles in standard position. The lines in the diagram are symmetrical about both the *x*-axis and the *y*-axis. Complete the diagram by determining both the degree measure and the radian measure at the end of each line.