

Trigonometry - Functions and Graphs Lesson #3: Trigonometric Ratios

Trigonometric Ratios

Note that some of the work in this lesson is a review of work covered in previous courses.

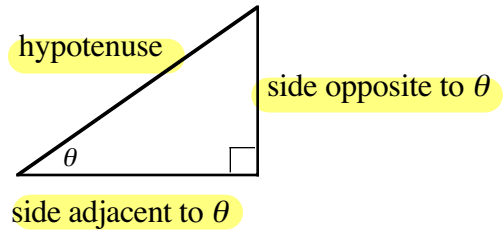
Primary Trigonometric Ratios

Complete the following:

sine ratio $\Rightarrow \sin \theta = \frac{O}{H}$

cosine ratio $\Rightarrow \cos \theta = \frac{A}{H}$

tangent ratio $\Rightarrow \tan \theta = \frac{O}{A}$



These ratios are called the **Primary Trigonometric Ratios** and can be remembered by the acronym **SOHCAHTOA**.

Reciprocal Trigonometric Ratios - the reciprocals of the primary trigonometric ratios.

cosecant ratio $\Rightarrow \csc \theta = \frac{1}{\sin \theta}$

secant ratio $\Rightarrow \sec \theta = \frac{1}{\cos \theta}$

cotangent ratio $\Rightarrow \cot \theta = \frac{1}{\tan \theta}$

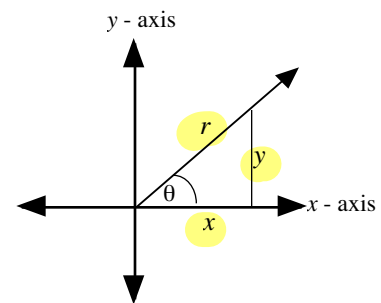


- We can remember the reciprocal from each of the primary trigonometric ratios by the fact that each “pair” has only one “co” prefix in it.
- The primary and reciprocal trigonometric ratios can be given in terms of x , y and r .



Use the diagram to write all the trigonometric ratios in terms of x , y and r .

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$
 $\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$ $\cot \theta = \frac{x}{y}$

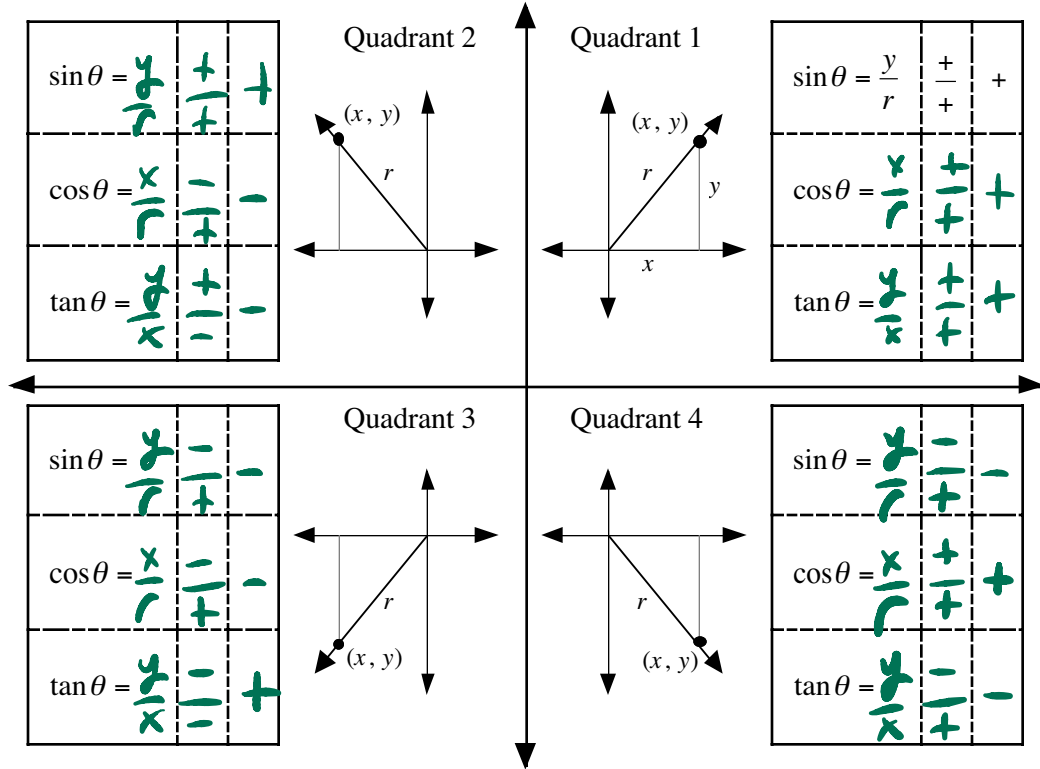


You should memorize these formulas.

Some students use a phrase like “seven yellow rabbits” to remember $\sin \theta = \frac{y}{r}$.

Determining the Sign of a Trigonometric Ratio

- a) In quadrant 1, draw the rotation angle θ in standard position and complete the table.
 b) Repeat for quadrants 2 - 4.



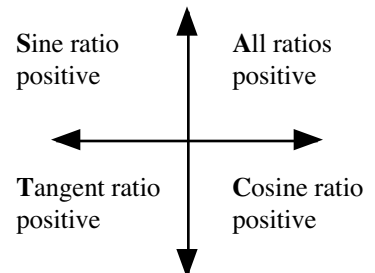
The reciprocal trigonometric ratios follow the same framework as their corresponding primary ratio.

- c) Complete the following statements using the information above.
- i) Sine ratios and cosecant ratios have **positive** values in quadrants 1 and 2.
 - ii) Cosine ratios and secant ratios have **positive** values in quadrants 1 and 4.
 - iii) Tangent ratios and cotangent ratios have **positive** values in quadrants 1 and 3.
 - iv) Sine ratios and cosecant ratios have **negative** values in quadrants 3 and 4.
 - v) Cosine ratios and secant ratios have **negative** values in quadrants 2 and 3.
 - vi) Tangent ratios and cotangent ratios have **negative** values in quadrants 2 and 4.

CAST Rule

The results can be memorized by:

- the **CAST** rule or
- by remembering to “Add Sugar To Coffee”





Class Ex. #2

Determine, without using technology, whether the given trigonometric ratios are positive or negative.

a) $\sin 280^\circ$

quad. IV
negative.

b) $\tan \frac{7\pi}{6}$

quad III
positive.

c) $\cos \frac{10\pi}{3}$

quad III
negative.

d) $\csc(-225^\circ)$

quad II
positive.

e) $\cot\left(-\frac{13\pi}{6}\right)$

quad IV
negative.

f) $\sec 1000^\circ$

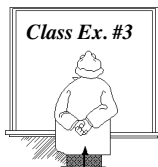
quad IV
positive.

Degree Mode and Radian Mode on a Calculator

Most calculators have the ability to calculate the values of sine ratios, cosine ratios, and tangent ratios, in degrees and in radians. To calculate the values of the reciprocal trigonometric ratios (cosecant ratios, secant ratios, and cotangent ratios), we need to use the reciprocal of the corresponding primary ratios.

For example, to calculate the value of $\csc \frac{5\pi}{6}$, set the calculator to radian mode, and do either of the following:

$$\csc \frac{5\pi}{6} = \frac{1}{\sin \frac{5\pi}{6}} = 2 \quad \text{or} \quad \sin \frac{5\pi}{6} = \frac{1}{2}, \quad \csc \frac{5\pi}{6} = \frac{1}{1/2} = 2$$



Class Ex. #3

Use a calculator to determine, to four decimal places where necessary, the values of the trigonometric ratios in Class Ex. #2.

a) $\sin 280^\circ$ **D**

-0.9848

b) $\tan \frac{7\pi}{6}$ **R**

0.5774

c) $\cos \frac{10\pi}{3}$ **R**

-0.5

d) $\sin 90^\circ$ **R**

0.8940

e) $\csc(-225^\circ)$ **D**

$\frac{1}{\sin(-225^\circ)}$
1.4142

f) $\cot\left(-\frac{13\pi}{6}\right)$ **R**

$\frac{1}{\tan\left(-\frac{13\pi}{6}\right)}$
-1.7321

g) $\sec 1000^\circ$ **D**

$\frac{1}{\cos 1000^\circ}$
5.7588

Expressing Trigonometric Ratios of an Angle in Terms of a Reference Angle

The trigonometric ratios for any angle are either the trigonometric ratios of the reference angle, or the negative of the trigonometric ratios of the reference angle.

Use the following procedure.

- i) Determine the sign of the ratio (positive or negative).
- ii) Determine the measure of the reference angle.
- iii) Combine i) and ii).

To write $\cos \frac{11\pi}{8}$ as the cosine of an acute angle using the above procedure, we have

- i) the sign is negative, ii) the reference angle = $\frac{3\pi}{8}$, and so iii) $\cos \frac{11\pi}{8} = -\cos \frac{3\pi}{8}$.

The result can be verified on a calculator.



Rewrite as the same trigonometric function of an acute angle.

a) $\cos \frac{7\pi}{4}$ **IV (+)** b) $\cot(-100^\circ)$ **III (+)** c) $\sin \frac{11\pi}{2}$

$\cos \frac{7\pi}{4} = \cos \frac{\pi}{4}$ $\cot(-100^\circ) = \cot 80^\circ$ $\sin \frac{11\pi}{2} = -\sin \frac{\pi}{2}$

Handwritten note: red circle with a cross, red arrow pointing to $\frac{\pi}{2}$

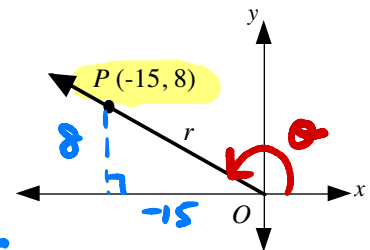
Complete Assignment Questions #1- #4

Determining Trigonometric Ratios from the a Point on the Terminal Arm



The point $P(-15, 8)$ lies on the terminal arm of a rotation angle θ in standard position.

- a) Mark the angle θ on the diagram.



- b) Calculate the exact length of $OP = r$.

$r^2 = 8^2 + (-15)^2$
 $r = \sqrt{64 + 225}$
 $r = \sqrt{289}$
 $r = 17$

- c) Use $x = -15$, $y = 8$ and r from b) to determine the exact values of the primary and reciprocal trigonometric ratios for angle θ .

$\sin = \frac{y}{r} = \frac{8}{17}$ $\csc = \frac{r}{y} = \frac{17}{8}$
 $\cos = \frac{x}{r} = \frac{-15}{17}$ $\sec = \frac{r}{x} = \frac{17}{-15}$
 $\tan = \frac{y}{x} = \frac{8}{-15}$ $\cot = \frac{x}{y} = \frac{-15}{8}$

#4-9

Using One Trigonometric Ratio to Determine Other Trigonometric Ratios
Class Ex. #6


Cot $A = \frac{\sqrt{5}}{2}$ and $\sin A$ is negative. Complete the following procedure to determine exact values, with rational denominators, for $\csc A$ and $\sec A$.

- a) Since the cotangent ratio is positive, and the sin ratio is negative, angle A must terminate in quadrant ____ .
- b) Since $\cot A = \frac{\sqrt{5}}{2} = \frac{x}{y}$, we know that the point $(-\sqrt{5}, -2)$ lies on the terminal arm of angle A in the third quadrant.
Sketch a diagram and draw the reference triangle illustrating the above information.
- c) Use $x^2 + y^2 = r^2$ to determine the value of r , and hence determine the exact values of $\csc A$ and $\sec A$.

Complete Assignment Questions #5 - #15

Assignment

1. In which quadrant(s) does the terminal arm of rotation angle θ lie if
 - a) $\sin \theta$ is negative?
 - b) $\sec \theta$ is positive?
 - c) $\csc \theta$ and $\tan \theta$ are both negative?
 - d) $\cot \theta$ is positive and $\csc \theta$ is negative?

2. Determine, without using technology, whether the given trigonometric ratios are positive or negative.
 - a) $\cos 181^\circ$
 - b) $\csc \frac{11\pi}{6}$
 - c) $\tan(-300^\circ)$
 - d) $\sin \frac{14\pi}{3}$
 - e) $\cot 560^\circ$
 - f) $\sec\left(-\frac{\pi}{4}\right)$

3. Find the value (to 4 decimal places where necessary) of

a) $\tan \frac{\pi}{4}$

b) $\cos (-382^\circ)$

c) $\sin \left(-\frac{2\pi}{3} \right)$

d) $\cot 30^\circ$

e) $\csc 60$

f) $\sec \left(-\frac{7\pi}{6} \right)$

4. Rewrite as the same trigonometric function of a positive acute angle.

a) $\sin 205^\circ =$ _____

b) $\cot \frac{3\pi}{5} =$ _____

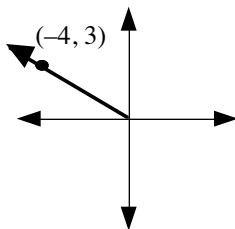
c) $\csc 107^\circ =$ _____

d) $\sec \left(-\frac{19\pi}{9} \right) =$ _____

e) $\cos 5\pi =$ _____

f) $\tan (-30^\circ) =$ _____

5. The point $(-4, 3)$ lies on the terminal arm of a rotation angle as shown. Determine the primary and reciprocal trigonometric ratios for the rotation angle. Express each answer as an exact value.



6. The point $(10, -24)$ lies on the terminal arm of an angle θ in standard position. Determine the exact values of $\sec \theta$ and $\csc \theta$.
7. Solve for the required ratios in each of the following. Express each answer as an exact value with a rational denominator.
- a) If $\tan \theta = \frac{\sqrt{2}}{4}$, and angle θ terminates in the first quadrant, determine $\cot \theta$, $\csc \theta$, and $\sec \theta$.
- b) If $\tan \theta = \frac{\sqrt{2}}{4}$, and angle θ terminates in the third quadrant, determine $\cot \theta$, $\csc \theta$, and $\sec \theta$.
8. If $\sin X = -\frac{1}{3}$ and $\cos X$ is positive, express $\cot X$ as an exact value.
9. $\cos A = -0.28$, where $\pi \leq A \leq \frac{3\pi}{2}$. Determine the exact value of $\csc A$.

**Multiple
Choice**

10. The point $\left(\frac{1}{5}, -\frac{1}{5}\right)$ lies on the terminal arm of an angle A in standard position. The exact value of $\sec A$ is

- A. $\sqrt{2}$ B. $-\sqrt{2}$
C. $\frac{\sqrt{2}}{25}$ D. $-\frac{\sqrt{2}}{25}$

Use the following information to answer the next question.

Consider the following trigonometric expressions.

- | | |
|-----------------------------|-----------------------------|
| I. $\cos(2\pi + x)$ | II. $\cos(2\pi - x)$ |
| III. $\cos(\pi - x)$ | IV. $\cos(-x)$ |

11. If $\cos x = A$, which of the following is not equal to A ?
- A. **III** only
B. **IV** only
C. **III** and **IV** only
D. some other combination of **I**, **II**, **III**, and **IV**
12. Without using technology, determine which of the following has a different sign from the others.
- A. $\tan 201^\circ$
B. $\csc(-72^\circ)$
C. $\sec 115^\circ$
D. $-\cot 79^\circ$
13. Without using technology, determine which of the following does not have the same value as $\cot 277^\circ$.
- A. $\cot(-83^\circ)$
B. $\cot(-263^\circ)$
C. $-\cot 263^\circ$
D. $-\cot 97^\circ$

Numerical Response

Use the following information to answer the next question.

Angles A , B , and C are rotation angles with the following properties.

- $\csc A = \csc \frac{\pi}{4}$ where $0 \leq A \leq 2\pi, A \neq \frac{\pi}{4}$
- $\cot B = \cot \frac{3\pi}{4}$ where $0 \leq B \leq 2\pi, B \neq \frac{3\pi}{4}$
- $\sec C = \sec \frac{8\pi}{5}$ where $0 \leq C \leq 2\pi, C \neq \frac{8\pi}{5}$

14. If the value of $A + B + C$ can be expressed in the form $k\pi$, then the value of k , to the nearest hundredth, is _____.

(Record your answer in the numerical response box from left to right.)

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15. The point $(4, -8)$ lies on the terminal arm of an angle θ . If the value of $\sin\theta + \sec\theta$ can be expressed in the form $k\sqrt{5}$, then the value of k , to one decimal place, is _____.

(Record your answer in the numerical response box from left to right.)

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Group Investigation

The following problems could be used as a lead-in to the next lesson. Use the blank pages at the back of the workbook to answer this group investigation.

- a) Sketch an angle of $\frac{\pi}{3}$ in standard position with the point $P(1, \sqrt{3})$ on the terminal arm. Without using technology, explain and carry out a strategy to determine the exact trigonometric ratios of three different angles greater than $\frac{\pi}{2}$ and less than 2π .
- b) Consider an angle A in standard position with $\sin A = -\frac{5}{13}$ and $0 \leq A \leq 2\pi$. Without using technology, explain and carry out a strategy to determine the exact values of $\cos A$ and $\tan A$.