Trigonometry - Functions and Graphs Lesson #4:
Determining Angle Measure from a Trigonometric Ratio

Review: Angle Measure in Degrees From Primary Trigonometric Ratios

In previous courses, we used the concepts of reference angle and sign of the trigonometric ratio to determine angle measures in degrees given a sine, cosine, or tangent ratio.

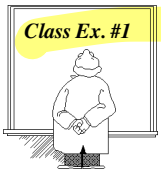
Use the following procedure to determine the angle measure between 0° and 360° given a trigonometric ratio.

Step 1: Determine the quadrant(s) the angle will be in by looking at the sign of the ratio.

Step 2: Determine the reference angle (always between 0° and 90°) and draw a rough sketch in the appropriate quadrant(s). To determine the reference angle, use [2nd] [sin] or [2nd] [cos] or [2nd] [tan] of the **absolute value** of the given quantity.

Step 3: Determine the rotation angle(s) using the reference angle and the quadrant(s).

- Always check the given domain to determine which quadrants are valid in the calculation. Sometimes the domain is restricted to, for example, $0° \le \theta \le 180°$, or $90° \le \theta \le 180°$.
- In the next unit, we will consider domains less than 0° or greater than 360°.



Complete the following to solve $\cos\theta = -0.5$, where $0° \le \theta \le 360°$.

$\cos\theta = -0.5$

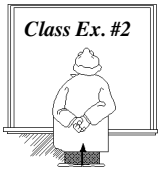
$\cos^{-1}(-0.5) =$

quadrants 2 and 3

reference angle = 60°

in quadrant 2, rotation angle = 120°

in quadrant 3, rotation angle = 240°

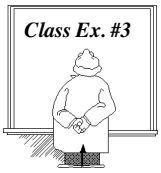


Given that $(\sin\theta)^2$ can be written as $\sin^2\theta$, solve the equation $\sin^2\theta = 0.5$ on the interval $0° \le \theta \le 360°$.

Angle Measure in Degrees From Reciprocal Trigonometric Ratios

Since there are no calculator keys for cosecant, secant, or cotangent, we must rewrite the reciprocal ratios in their primary form.

For example, to solve $\cot x = \sqrt{3}$, we rewrite this in the primary form $\tan x = \frac{1}{\sqrt{3}}$.



Solve $\cot x = \sqrt{3}$, $0° \le x \le 360°$.

$\frac{1}{\tan x} = \sqrt{3}$ $\quad \tan x = \frac{1}{\sqrt{3}}$ → I, III $\quad \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$ ref $\angle = 30°$

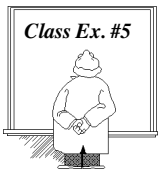
I $= 30°$, III $= 210°$

Determine the measure of x, to the nearest degree, where $0° \le x \le 360°$.

a) $\sec x = -1.2631$ $\quad$ **b)** $\csc x = 2.45$

$\cos x = \frac{1}{-1.2631}$ $\quad$ ref $\angle = 38°$ $\quad$ II → 142° $\quad$ III → 218°

$\sin x = \frac{1}{2.45}$ $\quad$ ref $\angle = 24°$ $\quad$ I → 24° $\quad$ II → 156°



Determine the measure of θ, to the nearest whole number, where $0° \le \theta \le 360°$.

a) $\cos \theta = 0$ $\quad$ **b)** $\csc \theta$ is undefined

$x = 0$ → 90°, 270°

$\frac{1}{\sin\theta} = \emptyset$ $\quad \sin\theta = 0$ $\quad y = 0$ → 0°, 180°, 360°

Complete Assignment Questions #1 - #4

Angle Measure in Radians From Trigonometric Ratios

A similar procedure can be used to determine angle measure in radians by setting the calculator mode to radian measure.

Step 1: Determine the quadrant(s) the angle will be in by looking at the sign of the ratio.

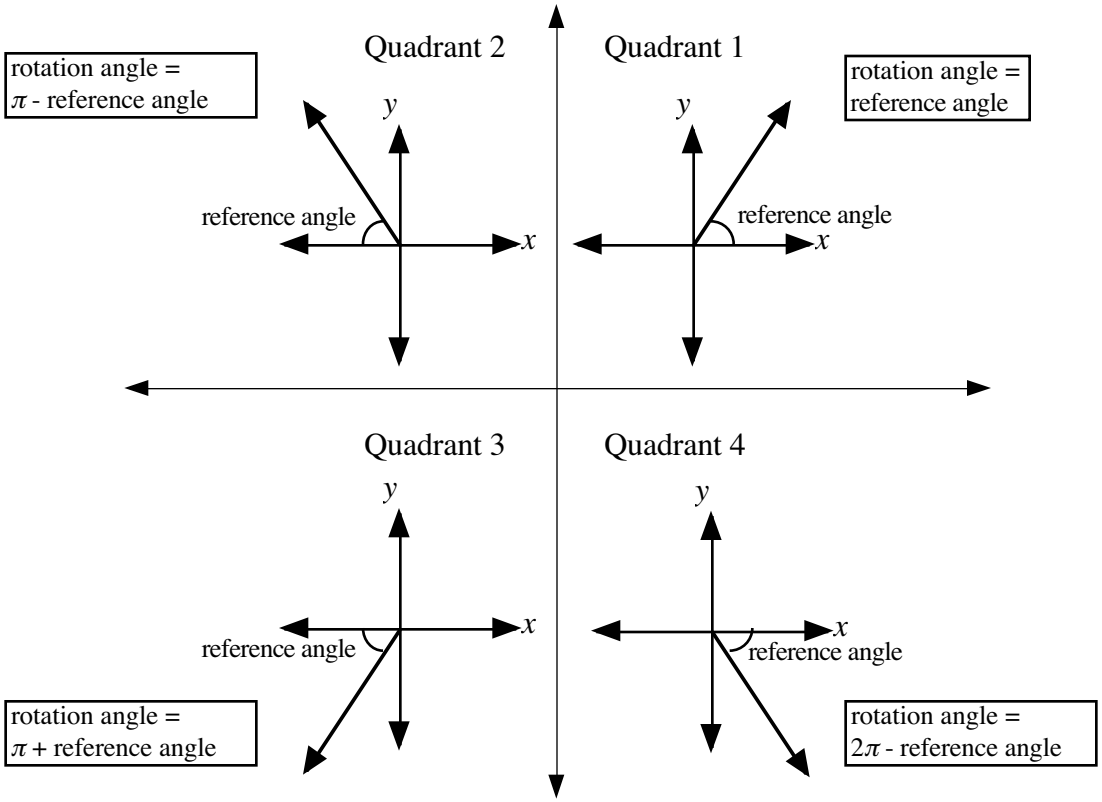
Step 2: Determine the reference angle (always between 0 and $\frac{\pi}{2}$ (approximately 1.57)).

Draw a rough sketch in the appropriate quadrant(s).
To determine the reference angle, use

[2nd] [sin] or [2nd] [cos] or [2nd] [tan]

of the **absolute value** of the given quantity.

Step 3: Determine the rotation angle(s) using the reference angle and the quadrant(s).



Class Ex. #6

In each case, determine the value(s) of θ to the nearest hundredth of a radian.

a) $\cos\theta = -0.5, 0 \le \theta \le 2\pi$

$\cos^{-1}(-0.5)$ ref $= 1.05$

II → $\pi - 1.05 = 2.09$

III → $\pi + 1.05 = 4.19$

b) $\cot\theta = -\frac{5}{4}, 0 \le \theta \le \pi$ *

$\tan\theta = -\frac{4}{5}$

$\tan^{-1}\left(-\frac{4}{5}\right) = \text{ref} = 0.67$

II → $\pi - 0.67 = 2.47$

#1-6 (a,c), 7-11