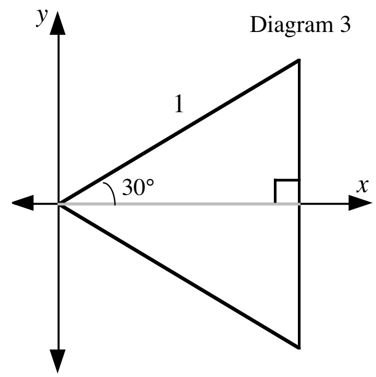
c) Diagram 3 shows an angle of 30° $\left(\text{or } \dfrac{\pi}{6} \text{ radians}\right)$ in standard position. The terminal arm has a length of 1 unit.

An equilateral triangle (congruent to the one in b)) is drawn whose equal sides are 1 unit. A horizontal altitude is drawn which divides the equilateral triangle into two congruent triangles.



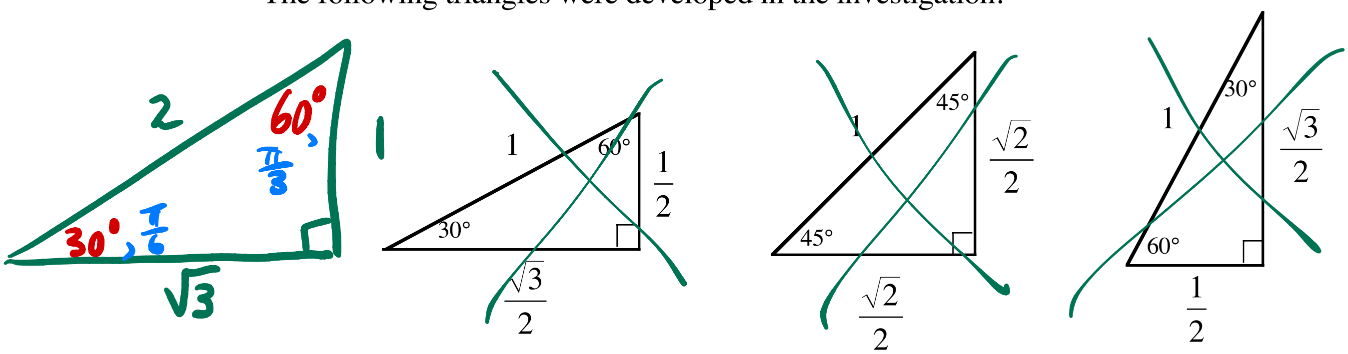
Complete:

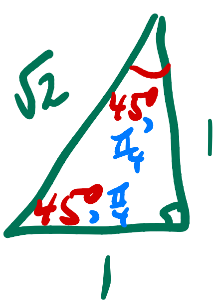
sin 30° = cos 30° = tan 30° =

$\sin \dfrac{\pi}{6} =$ $\cos \dfrac{\pi}{6} =$ $\tan \dfrac{\pi}{6} =$

Special Triangles

The following triangles were developed in the investigation.





a) Complete the following table.

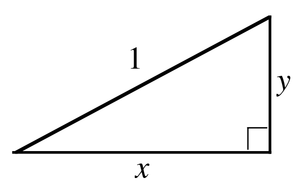
θ	30°	45°	60°
θ	$\dfrac{\pi}{6}$	$\dfrac{\pi}{4}$	$\dfrac{\pi}{3}$
$\sin\theta$			
$\cos\theta$			
$\tan\theta$			

b) If in each triangle the horizontal distance is x, the vertical distance is y, and the hypotenuse is 1, express the following trigonometric ratios in terms of x and/or y.

i) sine ratio

ii) cosine ratio

iii) tangent ratio



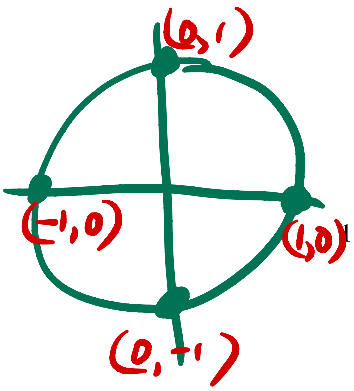
The Unit Circle

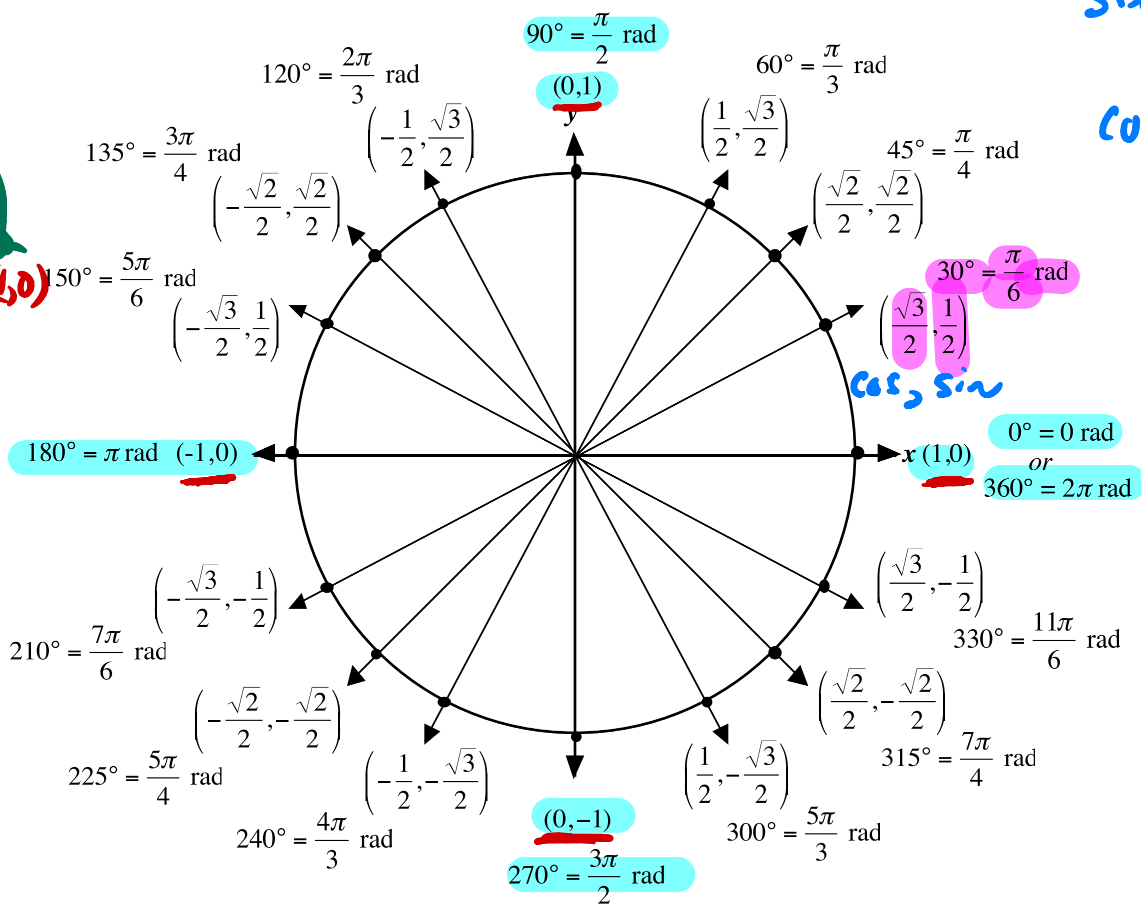
$r = 1$

The **unit circle** can be formed by reflecting the above diagram in the *x*-axis, in the *y*-axis, and in both the *x*-axis and the *y*-axis.

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$





The circle above, with a radius of one unit, is called the **unit circle** and it is important to understand how it works.

Recall the formulas $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$, and $\cot\theta = \frac{x}{y}$.

any point on the unit circle is $(\cos\theta, \sin\theta)$

- In the unit circle, where $r = 1$, we have:

  $\sin\theta = \underline{\ y\ }$ and $\cos\theta = \underline{\ x\ }$

- Every point on the unit circle has coordinates (x, y) which can be written as $(\cos\theta, \sin\theta)$

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Class Ex. #1

Use the unit circle to find the exact value of all the trigonometric ratios for a rotation angle of 300°. Give each answer with a rational denominator.

ref ∠ = 60° in IV

sin 300° = $-\frac{\sqrt{3}}{2}$ cos 300° = $\frac{1}{2}$ tan 300° = $-\sqrt{3}$

csc 300° = $-\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$ sec 300° = 2 cot 300° = $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

Class Ex. #2

Use the unit circle to find the exact value of

a) $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ **b)** $\cot\frac{5\pi}{3} = -\frac{\sqrt{3}}{3}$ **c)** tan 600° $= \sqrt{3}$ **d)** $\csc 3\pi$ $\frac{1}{\sin 3\pi} = \frac{1}{0}$ ∅

ref ∠ = $\frac{\pi}{4}$ quad. II ref ∠ = $\frac{\pi}{3}$ quad. IV ref ∠ = 60° quad. III

Class Ex. #3

Use a calculator to determine, to four decimal places, the coordinates of the point on the unit circle that corresponds to a rotation of $\frac{2}{5}\pi$.

$\left(\cos\frac{2\pi}{5}, \sin\frac{2\pi}{5}\right)$

$= (0.3090, 0.9511)$

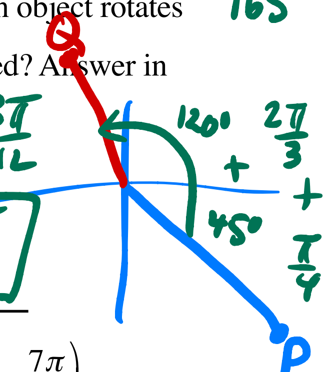
Class Ex. #4

$P\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ and $Q\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are two points on the unit circle. If an object rotates counterclockwise from point P to point Q, through what angle has it rotated? Answer in degrees and in radians.

cos, sin cos, sin

165°

$\frac{2\pi}{3} + \frac{\pi}{4} = \frac{8\pi}{12} + \frac{3\pi}{12} = \boxed{\frac{11\pi}{12}}$



∠P
ref ∠ = 45° → $\boxed{315°}$
IV = $\frac{\pi}{4}$ → $\boxed{\frac{7\pi}{4}}$

∠Q
ref ∠ = 60° → $\boxed{120°}$
II = $\frac{\pi}{3}$ → $\boxed{\frac{2\pi}{3}}$

Class Ex. #5

Without using a graphing calculator, determine the exact value of $\log_2\left(\cos\frac{7\pi}{4}\right)$.

→ ref ∠ = $\frac{\pi}{4}$ in IV

$\log_2\left(\frac{\sqrt{2}}{2}\right) = \log_2\left(\frac{2^{\frac{1}{2}}}{2^1}\right)$

$= \cancel{\log_2} 2^{-\frac{1}{2}} = \boxed{-\frac{1}{2}}$

Class Ex. #6

The point $A(-0.6157, -0.7880)$ lies on the unit circle. Determine the value of θ, where θ is the angle made by the positive x-axis and the line passing though A.

$\cos A = -0.6157$
$\sin A = -0.7880$

III ref ∠ = $\sin^{-1}(-0.7880) = 52°$

o
$180° + 52° = \boxed{232°}$

rads

ref ∠ = $\sin^{-1}(-0.7880) = 0.91$ $\pi + 0.91 = \boxed{4.05}$

$\boxed{\#2\text{-}8, 10, 12}$