Exponential and Logarithmic Functions Lesson #6:
Laws of Logarithms

Investigating the Product Law

a) Evaluate the following.

i) $\log_2 16 + \log_2 8$ $= 4 + 3 = 7$

ii) $\log_2 [(16)(8)]$ $= \log_2 128 = 7$

b) Evaluate the following.

i) $\log_3 27 + \log_3 3$ $= 3 + 1 = 4$

ii) $\log_3 [(27)(3)]$ $= \log_3 81 = 4$

c) Comment on the answers from a) and b).

the same.

Investigating the Quotient Law

a) Evaluate the following.

i) $\log_2 16 - \log_2 8$ $= 4 - 3 = 1$

ii) $\log_2 \frac{16}{8}$ $= \log_2 2 = 1$

b) Evaluate the following.

i) $\log_3 27 - \log_3 3$ $= 3 - 1 = 2$

ii) $\log_3 \frac{27}{3}$ $= \log_3 9 = 2$

c) Comment on the answers from a) and b).

the same.

Product and Quotient Laws of Logarithms

The above investigations are examples of the following laws.

$\log_a(M \times N) = \log_a M + \log_a N$	**The Product Law**
$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$	**The Quotient Law**

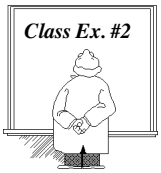
bases must be the same

Evaluate the following using the product law or quotient law.

a) $\log_2 12 - \log_2 3$

b) $\log_6 9 + \log_6 8 - \log_6 2$

$\log_6 \frac{(9)(8)}{(2)} = \log_6 36 = 2$



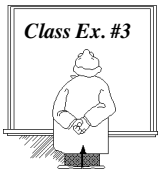
a) Use the laws of logarithms to write $\log_x 10 + \log_x 75 - (\log_x 2 + \log_x 3)$ as a single logarithm.

$\log_x 10 + \log_x 75 - \log_x 2 - \log_x 3$

$\log_x \left(\frac{750}{6}\right) = \boxed{\log_x 125}$

b) Evaluate a) if $x = 5$.

$\log_5 125 = \boxed{3}$



a) Use the laws of logarithms to write $\log_b 2 + \log_b 3 - \log_b 6 - \log_b 8$ as a single logarithm.

$\log_b \left(\frac{2 \cdot 3}{6 \cdot 8}\right) = \log_b \left(\frac{1}{8}\right)$

b) Evaluate a) if $b = 2$.

$\log_2 \left(\frac{1}{8}\right) = \boxed{-3}$

The expression $\log_2 x + \log_2 2x - \log_2 x^2 - \log_2 y$ is equivalent to

A. $2 + \log_2 y$

B. $1 + \log_2 y$

C. $2 - \log_2 y$

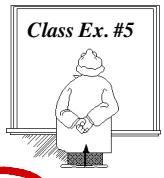
D. $1 - \log_2 y$

$\log_2 x + \log_2 2x - \log_2 x^2 - \log_2 y$

$\log_2 \left(\frac{2x^2}{x^2 y}\right) = \log_2 \left(\frac{2}{y}\right)$

$= \log_2 2 - \log_2 y$

$1 - \log_2 y.$



Determine the value of $3 \log_2 p - 3 \log_2 q$ if $\frac{p}{q} = 8$.

$3(\log_2 p - \log_2 q)$

$3 \log_2 p - 3 \log_2 q$

$3 \left(\log_2 \left(\frac{p}{q}\right)\right) = 3 \log_2 8 = 3 \cdot 3 = \boxed{9}$

$\log_2 p^3 - \log_2 q^3$

$\log_2 \left(\frac{p^3}{q^3}\right)$

$\log_2 \left(\frac{p}{q}\right)^3$

$\log_2 8^3$

$\log_2 (2^3)^3 = 3 \cdot 3 = \boxed{9}$

Complete Assignment Questions #1 - #4

Investigating the Power Law

a) By writing $2 \log x$ as $\log x + \log x$, show that $2 \log x = \log x^2$.

$2\log x = \log x^2 \qquad \log x + \log x = \log x^2$

b) Prove that $3 \log_2 a = \log_2 a^3$.

$\log_2 a + \log_2 a + \log_2 a = \log_2 a^3$

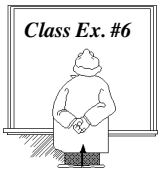
c) Write an expression equivalent to $a \log_b c$.

$a \log_b c = \log_b c^a$

The Power Law of Logarithms

The above investigation is an example of the power law of logarithms.

$\log_a M^n = n \log_a M$	**The Power Law**



Without using a calculator, evaluate each of the following.

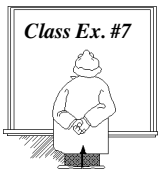
a) $\log_4 16^{12}$

$12 \log_4 16$

$12 \cdot 2 = 24$

b) $\log_{10} 10^{21}$

$= 21$



Simplify the following.

a) $\log_6 6^n$

n

b) $6^{\log_6 n}$

n

Class Ex. #7 is an example of the following logarithmic identities:

$$\log_b b^n = n \qquad \text{and} \qquad b^{\log_b n} = n$$

These identities follow from the fact that the logarithmic and exponential functions are inverses.

Complete Assignment Questions #5 - #14

#1-6 (a, c, e...)