Exponential and Logarithmic Functions Lesson #8:
Graphing Logarithmic Functions

Exploring the Value of b in $y = \log_b x$

We will investigate how changing the value of b affects the graph of $y = \log_b x$.
Notice that every graph of this form must pass through the point $(1, 0)$ because $\log_b 1 = 0$.

Part 1 The graph of $y = \log_3 x$ is shown. The graph passes through the point $(3, 1)$ because $\log_3 3 = 1$.

a) In each of the following, complete the statement and sketch the graph on the grid.
Use a graphing calculator with window format x:[–1, 11, 1] y:[–4, 4, 1].

i) $y = \log_{10} x$ passes through (10, 1).

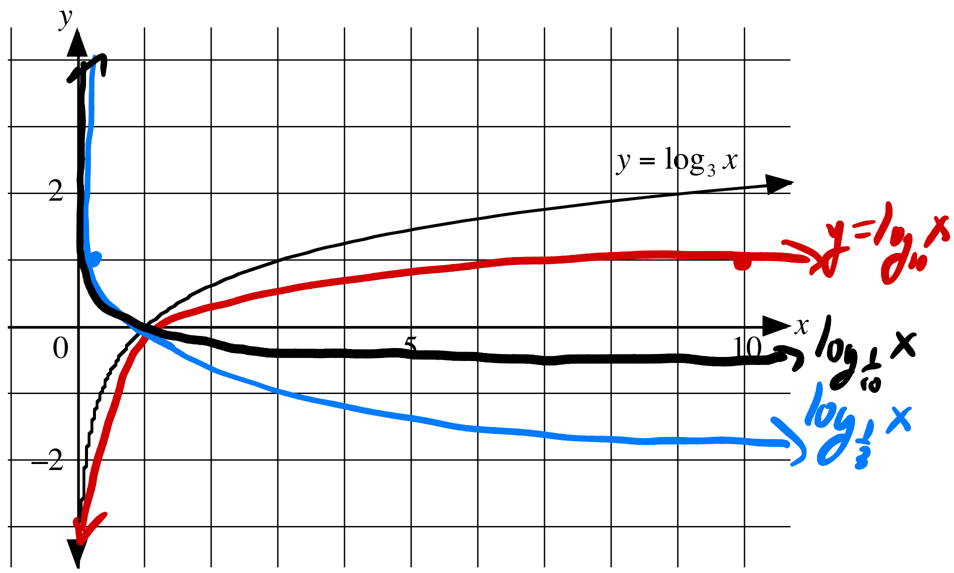
write as

ii) $y = \log_{\frac{1}{3}} x$ passes through ($\frac{1}{3}$, 1).

$\frac{\log x}{\log\left(\frac{1}{3}\right)}$

iii) $y = \log_{\frac{1}{10}} x$ passes through ($\frac{1}{10}$, 1).

$\frac{\log x}{\log\left(\frac{1}{10}\right)}$



b) Without using a graphing calculator, make a sketch of the graphs of the following and verify with a graphing calculator.

i) $y = \log_5 x$ **ii)** $y = \log_{\frac{1}{5}} x$

c) Complete the table.

Function	Domain	Range	x-int	y-int	Asymptote	x-value when $y = 1$
$y = \log_3 x$	$x > 0$	$y \in \mathbb{R}$	$(1, 0)$	none	$x = 0$	3
$y = \log_{10} x$						10
$y = \log_{\frac{1}{3}} x$						$\frac{1}{3}$
$y = \log_{\frac{1}{10}} x$						$\frac{1}{10}$
$y = \log_b x$						b

d) How do the graphs of $y = \log_3 x$ and $\log_{\frac{1}{3}} x$ compare with each other?

reflections on the x-axis

e) How do the graphs of $y = \log_{10} x$ and $\log_{\frac{1}{10}} x$ compare with each other?

reflections on the x axis.

f) Complete the following statement.

"The graph of $y = \log_{\frac{1}{b}} x$ is reflection on the x-axis of the graph of $y = \log_b x$."

g) In the transformation unit, the replacement for reflection in the x-axis is $y \to -y$. Starting with $y = \log_b x$, make this replacement to determine the equation of the graph reflected in the x-axis.

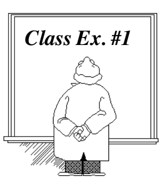
$y = \log_b x$

$-y = \log_b x \to y = -\log_b x$

We now have two equations for the graph of $y = \log_b x$ reflected in the x-axis. Hence

$$\boxed{\log_{\frac{1}{b}} x = -\log_b x}$$

$\log_{\frac{1}{b}} x = \log_{b^{-1}} x$



a) If $\log_4 x = 8$, state the value of $\log_{\frac{1}{4}} x$. $= -\log_4 x = \boxed{-8}$

b) Prove the result in a) by converting to exponential form.

$\log_4 x = 8$

$4^8 = x$

$\log_{\frac{1}{4}} x = v$

$\frac{1}{4}^v = x$

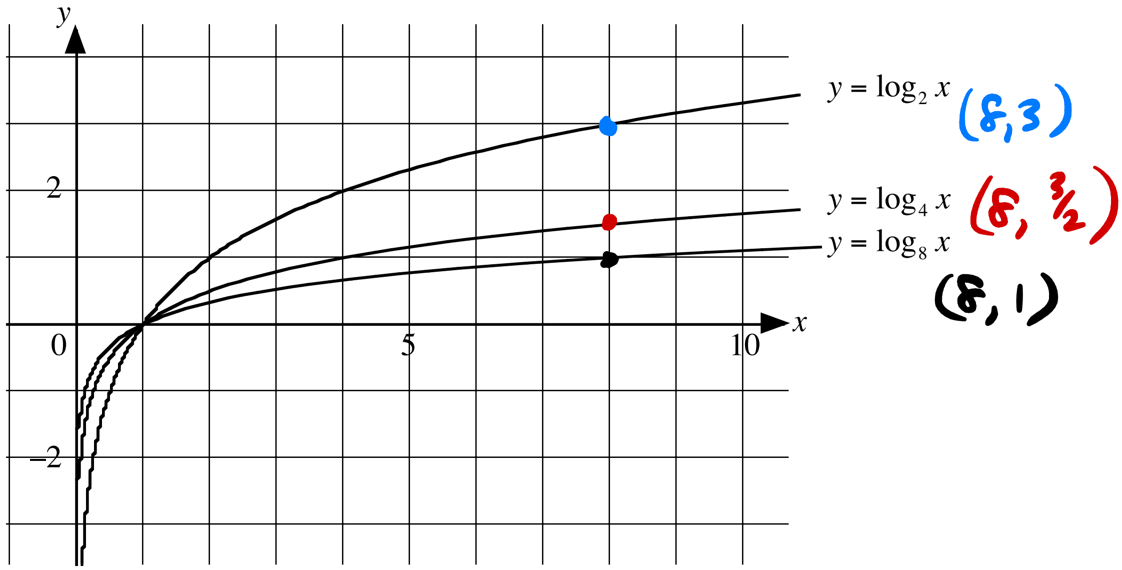
$\frac{1}{4}^v = 4^8$

$4^{-v} = 4^8$

$-v = 8$

$\therefore \boxed{v = -8}$

Part 2 The partial graphs of $y = \log_2 x$, $y = \log_4 x$, and $y = \log_8 x$ are shown.



a) Complete the following statements. **i)** $y = \log_2 x$ passes through (8 , 3).

ii) $y = \log_4 x$ passes through (8, $\frac{3}{2}$). **iii)** $y = \log_8 x$ passes through (8, 1).

b) The graph of $y = \log_4 x$ is a vertical stretch of the graph of $y = \log_2 x$ about the x-axis. State the stretch factor and complete the statement $\log_4 x = \frac{1}{2}\log_2 x$.

c) The graph of $y = \log_8 x$ is a vertical stretch of the graph of $y = \log_2 x$ about the x-axis. State the stretch factor and complete the statement $\log_8 x = \frac{1}{3}\log_2 x$.

d) i) Use the results above to determine the transformation which would map the graph of $y = \log_2 x$ to the graph of $y = \log_{64} x$. $= \frac{1}{6}\log_2 x$

ii) Complete the statement $\log_{64} x = \frac{1}{6}\log_2 x$. $\log_{64} x = \log_{2^6} x = \frac{1}{6}\log_2 x$

Note

The above is an example of the general rule.

> The transformation which maps $y = \log_b x$ to $y = \log_{b^n} x$ is a vertical stretch about the x-axis by a factor of $\frac{1}{n}$, so $\log_{b^n} x = \frac{1}{n}\log_b x$

Class Ex. #2

a) Describe the transformation which would map the graph of

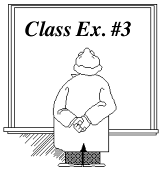
i) $y = \log_3 x$ to the graph of $y = \log_{81} x$ **ii)** $y = \log_{16} x$ to the graph of $y = \log_4 x$

vertical comp. by a factor of $\frac{1}{4}$ vert. exp by a factor of 2

b) Complete the statements: **i)** $\log_{81} x = \frac{1}{4}\log_3 x$ **ii)** $\log_4 x = 2\log_{16} x$

$\log_{3^4} x$ $\log_{16^{\frac{1}{2}}} x$

Class Ex. #3



a) If $\log_5 x = 6$, state the value of $\log_{125} x$. $= \log_{5^3} x = \frac{1}{3}\log_5 x = \frac{1}{3}(6) = \boxed{2}$

b) Prove the result in a) by converting to exponential form.

$\log_5 x = 6$
$x = 5^6$

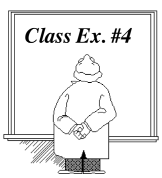
$\log_{125} x = v$
$125^v = 5^6$
$5^{3v} = 5^6$
$3v = 6 \Rightarrow \boxed{v = 2}$

Complete Assignment Questions #1 - #5

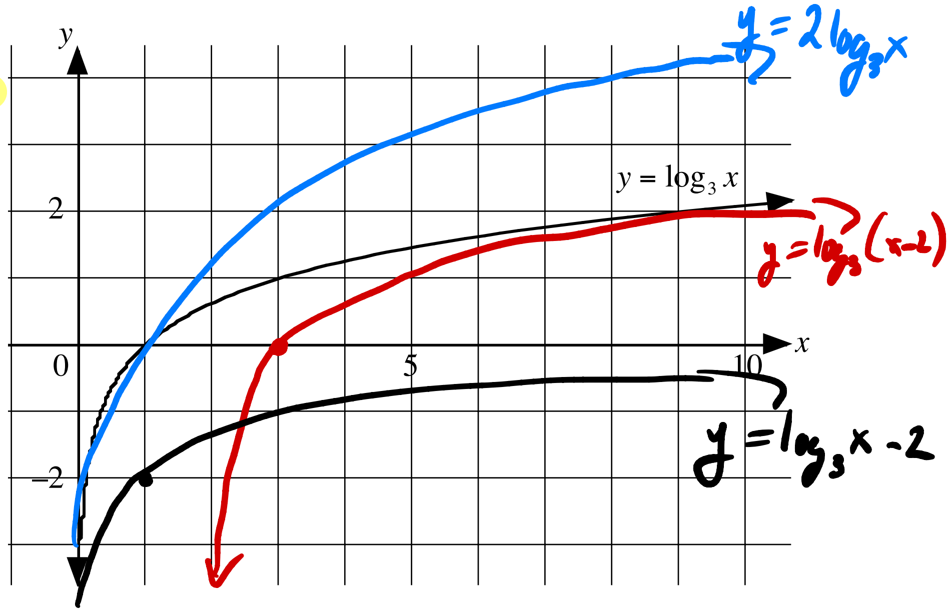
Further Transformations of Logarithmic Functions

We use the knowledge learned in *Transformations* to compare the graph of $y = \log_c x$ to the graph of $y = a\log_c b(x - h) + k$. We use the letter c to represent the base of the logarithm to distinguish it from the letter b which is associated with the horizontal stretch.

Class Ex. #4



The graph of $y = \log_3 x$ is shown.



a) Write the transformation associated with each of the following and sketch the graph on the grid.

i) $y = 2\log_3 x$
$y \to \frac{1}{2}y$
v.e. by a factor of 2

ii) $y = \log_3(x - 2)$
$x \to x - 2$
h.t. 2 units right

iii) $y = \log_3 x - 2$
$y \to y + 2$
v.t. 2 units down.

b) Do any of the above transformations result in a change in the original domain or range?

no change for range but ii. changed domain to $x > 2$

c) State the equation of the asymptote for each part of a).

i.) $x = 0$ ii.) $x = 2$ iii.) $x = 0$

$\boxed{\#1\text{–}4, 6, 7}$