Applications of Exponential and Logarithmic Functions
Lesson #1: Solving Exponential Equations

Overview

In this unit, we will learn how to solve exponential and logarithmic equations and apply this to real life problems involving exponential growth and decay, including compound interest, loans, investments, half-life, etc. We will also solve problems involving logarithmic scales, such as the Richter scale, the pH scale, and the Bell scale.

Review *Solving Exponential Equations with a Common Base*

In the last unit we solved exponential equations involving a common base.

Solve the following exponential equations by converting to a common base.

a) $4^x = 2^{x+3}$

$(2^2)^x = 2^{x+3}$

$2^{2x} = 2^{x+3}$

$2x = x+3$

$\boxed{x=3}$

b) $27^{2x-1} = 9(3)^{2x-1}$

$(3^3)^{2x-1} = 3^2 \cdot 3^{2x-1}$

$3^{6x-3} = 3^{2x+1}$

$6x-3 = 2x+1$

$4x = 4$

$\boxed{x=1}$

Solving Exponential Equations without using a Common Base

Consider the exponential equation $2^x = 50$.

a) Explain why the solution cannot be determined by the above method.

no rational common base.

b) i) Determine, by considering powers of 2, the two integers between which the solution must lie.

$2^5 = 32$ $\boxed{\approx 5.6}$ $2^6 = 64$

ii) Estimate the solution to the nearest tenth.

c) The equation $2^x = 50$ can be solved by taking the logarithm (base 10) of each side and solving the resulting equation.

Complete the work started below and give
i) the exact solution and, **ii)** an approximation to the nearest tenth.

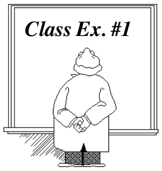
$2^x = 50$

$\cancel{\log 2^x = \log 50}$

$\cancel{x \log 2 = \log 50}$

$\log_2 50 = x$

$\boxed{\dfrac{\log 50}{\log 2}} = x \approx 5.6$



Solve the following equations, expressing the solution

i) as an exact value in the form $\dfrac{\log M}{\log N}$ **ii)** to three decimal places

a) $4^x = 12$

$\log_4 12 = x$

$x = \boxed{\dfrac{\log 12}{\log 4}} \approx \boxed{1.792}$

b) $8^x + 4 = 40$

$8^x = 36$

$\log_8 36 = x$

$x = \boxed{\dfrac{\log 36}{\log 8}} \approx \boxed{1.723}$

c) $6^{3x} = 3^{2x-1}$

take the common log of both sides.

$\log 6^{3x} = \log 3^{2x-1}$

$3x \log 6 = (2x-1)\log 3$

$3x \log 6 = 2x \log 3 - \log 3$

$3x \log 6 - 2x \log 3 = -\log 3$

Factor off "x"

$\dfrac{x\cancel{(3\log 6 - 2\log 3)}}{\cancel{3\log 6 - 2\log 3}} = \dfrac{-\log 3}{3\log 6 - 2\log 3}$

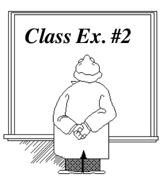
$x = \dfrac{\log 3^{-1}}{\log 6^3 - \log 3^2} = \boxed{\dfrac{\log\left(\frac{1}{3}\right)}{\log 24}} \approx \boxed{-0.346}$

d) $2(3)^{x-2} = 7^x$

$\log 2(3)^{x-2} = \log 7^x$

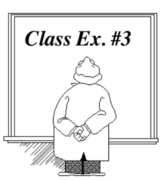
$\log 2 + \log 3^{x-2} = \log 7^x$

$\log 2 + (x-2)\log 3 = x \log 7$



In simple cases, such as $2^x = 50$ or $4^x = 12$, the equation can also be solved by converting to logarithmic form. For example $2^x = 50 \Rightarrow x = \log_2 50 = \dfrac{\log 50}{\log 2}$.

Solve $4^x = 12$ by converting to logarithmic form.



According to Statistics Canada, in the 21st century, Canada's population has been growing at an average annual rate of 1.0%. The exponential equation $P = P_o(1.01)^n$ can be used to predict the population, P, n years after a known population P_o.
(http://www.statcan.gc.ca/pub/91-003-x/2007001/4129907-eng.htm)

If the Canadian population at the beginning of 2001 was approximately 30 million, estimate in which year the Canadian population will reach 50 million assuming the same average annual rate of growth.

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