Applications of Exponential and Logarithmic Functions
Lesson #2: Applications in Finance

Investing Money

If you deposit money in a financial institution, such as a bank, you are in effect lending money to the bank. In exchange the bank pays you interest. There are two types of interest: **simple interest** and **compound interest**.

Simple Interest

Simple Interest is usually applicable to short term investments of one year or less or to longer term investments where the annual interest is paid to the investor and not reinvested.

a) If you invest \$500 which earns interest at a rate of 6% per year, how much interest would you earn in:

i) one year **ii)** half a year **iii)** one month?

b) If r is the annual interest rate (expressed as a decimal) and \$$P$ is the initial investment, calculate how much interest would be earned in:

i) one year **ii)** half a year **iii)** t years.

The formula to calculate simple interest is $\boldsymbol{I = Prt}$

where

I represents the amount of interest
P represents the principal (the initial investment)
r represents the annual rate of interest - expressed as a decimal
t represents the time in years for which the money is invested

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Millie invests in \$2350.00 at 7% per year for six months. Calculate, after six months,

a) the simple interest on Millie's investment

$I = (2350)(.07)(0.5)$
$= \$82.25$

b) the value of Millie's investment

$V = P + I$
$= 2350 + 82.25$
$= \$2432.25$

Compound Interest

In simple interest the principal at the beginning of the second year is the same as the principal at the beginning of the first year.

In compound interest the interest earned during the first year is added to the original principal to form a new principal.

To understand the comparison between simple interest and compound interest, do the investigation on the next two pages.

In the previous explorations, interest is compounded on an annual basis. In practice, compounding can take place over any period of time, eg semi-annually, monthly, daily, continuously, etc.

Compound Interest Formula

The formula which can be used to calculate compound interest is an exponential function of the form $y = ab^x$, where the domain of the function is discrete (not continuous).

$$A = P(1 + i)^n$$

where, A represents the final amount
P represents the initial principal
i represents the interest rate per compounding period
n represents the number of compounding periods.

- Note that i does **NOT** always represent the annual interest rate.
- Note that n does **NOT** always represent the number of years.

$1000 is invested for 5 years at an annual interest rate of 6%. Complete the table to calculate the final value of the investment if interest is compounded according to the period of time given in the table.

Compounding Period	*Number of Compounding Periods Per Year*	*Total Number of Compounding Periods*	*Interest Rate per Compounding Period*	*Formula A =*	*Amount*
Annually	1	5	.06	$A = 1000(1+.06)^5$	$1338.23
Semi-Annually	2	10	$\frac{.06}{2} = .03$	$A = 1000(1.03)^{10}$	$1343.92
Quarterly	4	20	$\frac{.06}{4} = .015$	$A = 1000(1.015)^{20}$	$1346.86
Monthly	12	60	$\frac{.06}{12} = .005$	$A = 1000(1.005)^{60}$	$1348.85

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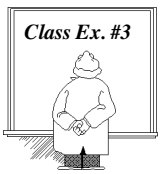


$7000 is invested in a 6 year GIC compounded quarterly at a rate of 5% per annum. Determine the value of the investment at the end of the term.

$A =$??
$P =$ 7000
$i =$ $.05/4 = .0125$
$n =$ $6 \times 4 = 24$

$A = 7000(1.0125)^{24}$
$= \$9431.46$

Christine invested $2500 for 4 years compounded semi-annually and received $843.26 interest. What was the annual rate of interest?

$A = 3343.26$
$P = 2500$
$i = ??$
$n = 8$

$A = P(1+i)^n$

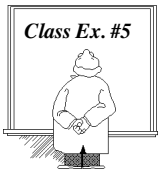
$\frac{3343.26}{2500} = \left(1+\frac{i}{2}\right)^8$

$\sqrt[8]{\frac{3343.26}{2500}} = 1+\frac{i}{2}$

$\left[\sqrt[8]{\frac{3343.26}{2500}} - 1\right] \cdot 2 = i$

$i = .074$

$i = 7.4\%$



Barbara invests $8000 in an account which pays compound interest of 6% per annum compounded monthly. How long would it take, in years and months, for her investment to double in value ?

$A =$
$P =$
$i =$
$n =$

Borrowing Money

When an amount of money is borrowed, interest is charged for the use of that money for a certain fixed period of time. If the loan is paid off in one payment at the end of the loan period, then the compound interest formula can be used.

If repayments are made on a regular basis during the period of the loan, the compound interest formula cannot be used.

Andrea borrows $7500 from her parents to buy new car. Her parents charge her interest at the rate of 4% p.a. compounded quarterly. When she pays off the loan, she has to pay $785 interest. What was the length of the loan?

$A = 8285$
$P = 7500$
$i = .04/4 = .01$
$n = ??$

$A = P(1+i)^n$

$\frac{8285}{7500} = \frac{7500(1.01)^n}{7500}$

$\frac{8285}{7500} = 1.01^n$

$\log_{1.01}\left(\frac{8285}{7500}\right) = n$

$\frac{\log\left(\frac{8285}{7500}\right)}{\log 1.01} = n$

$10 \text{ quarters} = n$

length of loan is 30 months or 2.5 yrs

Complete Assignment Questions #1 - #11

#1-8

Assignment

1. Calculate the simple interest in each case.

a) $740 is invested at 6% per annum for six months.

b) $1500 is invested at 8%/a for 3 months.