Applications of Exponential and Logarithmic Functions
Lesson #3: Applications of Exponential Growth or Decay

Review

- An **exponential function** is a function whose equation is of the form

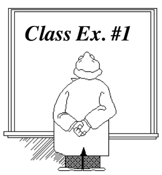
$$y = ab^x \quad \text{where } a \neq 0, b > 0, x \in R$$

- For $a > 0$,
  - when $b > 1$, the function represents a growth function.
  - when $0 < b < 1$, the function represents a decay function .

Writing an Equation Using $y = ab^x$

There are many applications of exponential functions in real life. In some cases, the function $y = ab^x$ can be written in "disguised" form. For example, $A = P(1 + i)^n$ is an exponential function whose base is $1 + i$. In this lesson, we will meet further real life applications of exponential growth and decay.

We can use variations of the formula $y = ab^x$ (such as $A = P(1 + i)^n$) to solve problems involving population growth, growth of bacteria, radioactive decay etc.



In 2012 the university population of a country was 160 000 and was increasing at an annual rate of 4.5%.

a) If the function representing the population is of the form $y = ab^x$, state the values for a and b.

$a = 160000 \qquad b = 1 + .045 = 1.045$

b) Write an equation to represent the university population, P, of the country as a function of the number of years, n, since 2012.

$P = 160000(1.045)^n$

c) Determine the population in the year 2015.

$n = 3$

$P = 160000(1.045)^3$

$= \boxed{182587}$

d) If the population continues to grow at this rate, determine the number of years, to the nearest year, for the population to double from its 2012 size.

$y = ab^x$

$\frac{320000}{160000} = \frac{160000(1.045)^n}{160000}$

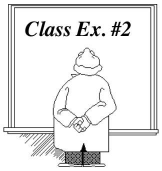
$2 = 1.045^n$

$\log_{1.045} 2 = n$

$n = \frac{\log 2}{\log 1.045}$

$\approx 15.7....$

$\boxed{16 \text{ yrs}}$



The number of fish in a lake is decreasing by 5% each year as a result of overfishing.

a) If the number of fish present is an exponential function of time, state the base of the exponential function.

$b = 1 - .05 = 0.95$

b) Write an equation to represent the number of fish present after *t* years. Use N_0 to represent the initial population and $N(t)$ to represent the final population.

$N(t) = N_0 (0.95)^t$

c) If there were 2500 fish present in June 2012, how many would you expect to be present in June 2017?

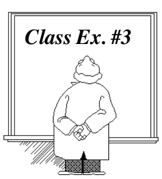
$N(t) = 2500(0.95)^5$
$= \boxed{1934}$

d) How many years, to the nearest tenth, would it take for the fish population to reduce to half of the number in June 2012?

$N(t) = N_0 (0.95)^t$
$0.5 = (0.95)^t$
$\log_{0.95} 0.5 = t$

$\frac{\log 0.5}{\log 0.95} = t$

$t = 13.5$ yrs



The intensity, I_0, of a light source is reduced to I after passing through d metres of a fog, according to the formula $I = I_0 e^{-0.12d}$. At what distance, to the nearest hundredth of a metre, will the intensity be reduced to one quarter of its original value?

Complete Assignment Questions #1 - #7