When attempting to verify the solution $x = -2$ in the previous example, we reached the situation of the logarithm of a negative number.

We know that logarithmic expressions are only defined for positive values of the argument. So in this case, $x = -2$ is an **extraneous solution**.

Notice that it is not the fact x is negative that makes the solution extraneous. It is the fact that the argument of the logarithm is negative that makes the solution extraneous.

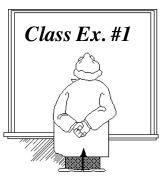
Solving Logarithmic Equations using the Laws of Logarithms

A logarithmic expression is defined only for positive values of the argument.

When we solve a logarithmic equation, it is essential to verify that the solutions do not result in the logarithm of a negative number. Solutions that would result in the logarithm of a negative number are called **extraneous**, and are not valid solutions.

Values obtained by solving logarithmic equations may be extraneous and must be verified. There are two verifications required when the solution is replaced in the original equation:

- All the arguments of the logarithms must be positive.
- If the base is variable, it too must be positive.

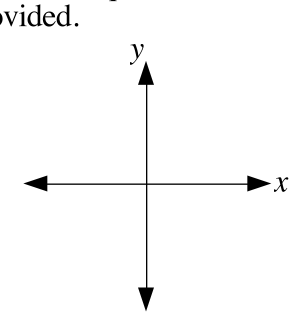


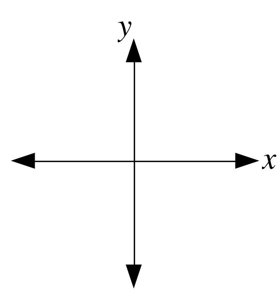
a) Solve the following logarithmic equations.

i) $\log x^2 = \log 16$

ii) $2 \log x = \log 16$

b) Solve the equations in a) using a graphing technique. Illustrate the solutions on the grids provided.





c) Explain why the solutions to the logarithmic equations are not the same even though $2 \log x$ can be written as $\log x^2$.

Class Ex. #2

Solve for x in the following equations.

* one log per side maximum *

a) $\log_4(x + 1) - \log_4(2x - 3) = \log_4 8$

$$\log_4\left(\frac{x+1}{2x-3}\right) = \log_4 8$$

$$\frac{x+1}{2x-3} = \frac{8}{1}$$

$$x+1 = 16x-24$$

$$\frac{25}{15} = \frac{15x}{15}$$

$$\boxed{5/3 = x}$$

b) $\log_6(x + 4) = 1 - \log_6(x + 3)$

$$\log_6(x+4) + \log_6(x+3) = 1$$

$$\log_6[(x+4)(x+3)] = 1$$

$$6 = x^2 + 7x + 12$$

$$0 = x^2 + 7x + 6$$

$$0 = (x+6)(x+1)$$

$$\boxed{x = \cancel{-6}, -1}$$

ext.

Class Ex. #3

Solve $2 \log_3 x - \log_3(x + 3) - 3 = 0$ to two decimal places.

$$2\log_3 x - \log_3(x+3) = 3$$

$$\log_3 x^2 - \log_3(x+3) = 3$$

$$\log_3\left(\frac{x^2}{x+3}\right) = 3$$

$$3^3 = \frac{x^2}{x+3}$$

$$(x+3)\left[27 = \frac{x^2}{\cancel{x+3}}\right]$$

$$27x + 81 = x^2$$

$$0 = x^2 - 27x - 81$$

$$x = \frac{27 \pm \sqrt{729 - 4(1)(-81)}}{2(1)}$$

$$= \frac{27 \pm \sqrt{1053}}{2} \approx \boxed{29.72} \ \& \ \cancel{-2.72}$$

ext.

Complete Assignment Questions #1 - #6

Solving More Complex Logarithmic Equations

Class Ex. #4

Solve the equation $\log_2(\log_x(3x + 4)) = 1$ by twice converting to exponential form. Remember to check for extraneous solutions.

$$\text{let } n = \boxed{\log_x(3x+4) = 2}$$

$$\log_2 n = 1$$

$$2^1 = n$$

$$n = 2$$

$$x^2 = 3x + 4$$

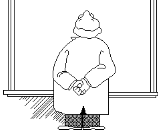
$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$\boxed{x = 4, \cancel{-1}}$$

ext.

Class Ex. #5



Solve and verify.

a) Solve the equation $\log x^2 - 3\log x = 10$ by using the laws of logarithms.

$2\log x - 3\log x = 10$

$-\log x = 10$

$\log_{10} x = -10$

$x = 10^{-10}$ or $\frac{1}{10^{10}}$

b) Explain why the equation $(\log x)^2 - 3\log x = 10$ cannot be solved using the laws of logarithms.

$(\log x)(\log x)$

c) Solve the equation in b) using the following steps:

1. Replace $\log x$ by a new variable A.
2. Solve for A.
3. Replace A by $\log x$ and solve for x.
4. Check for extraneous solutions.

let $A = \log x$

$A^2 - 3A = 10$

$A^2 - 3A - 10 = 0$

$(A-5)(A+2) = 0$

$A = 5, -2$

$\log x = 5$

$x = 10^5$

$= \boxed{100\,000}$

$\log x = -2$

$x = 10^{-2}$

$= \boxed{\frac{1}{100}}$

d) A student is solving the logarithmic equation $(\log x)^2 - \log x^3 = 0$. Her work is shown.

$3\log x$

$(\log x)^2 - \log x^3 = 0$ Let $A = \log x$.

$A^2 - A^3 = 0$

$A^2(1 - A) = 0$

$A = 0$ or $A = 1$

$\log x = 0$ or $\log x = 1$

$x = 10^0$ or $x = 10^1$

$x = 1, 10 \Rightarrow$ No extraneous solutions.

$A^2 - 3A = 0$

$A(A-3) = 0$

$A = 0$ $A = 3$

$\log x = 0$

$10^0 = x$

$\boxed{x = 1}$

$\log x = 3$

$10^3 = x$

$\boxed{x = 1000}$

There is an error in the student's work because $x = 10$ does not satisfy the original equation. Identify her error, and provide the correct solution in the space above.

Complete Assignment Questions #7 - #12

$\boxed{\#1\text{-}4, 8, 9}$