Applications of Exponential and Logarithmic Functions
Lesson #5: Logarithmic Scales and Applications

Earthquakes

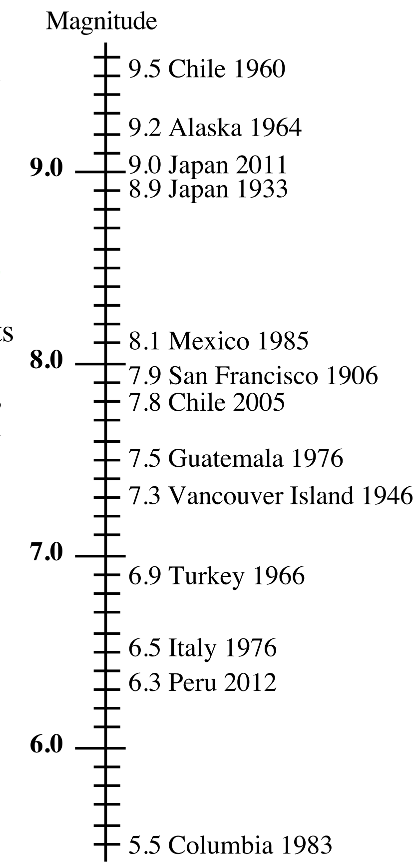
The Richter scale, named after the American seismologist Charles Richter (1900-85), is used to measure the magnitude of an earthquake. The magnitude of an earthquake is a measure of the amount of energy released. It is determined from the logarithm of the amplitude of waves recorded by seismographs.

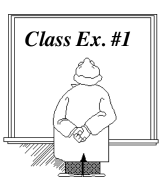
The Richter scale is logarithmic – a difference in one unit in magnitude corresponds to a factor of ten difference in intensity. This means that each whole number step represents a ten-fold increase in intensity. Therefore a magnitude 9 earthquake is ten times larger than a magnitude 8 earthquake, one hundred times larger then a magnitude 7 earthquake, and one thousand times larger than a magnitude 6 earthquake.

A magnitude of 1.0 on the Richter Scale is equivalent to a large blast at a construction site using approximately 14 kg of TNT. It is 10 times as intense as the zero reference point. An earthquake with magnitude 2.0 is 10^2 times as intense as the zero reference point, etc.

Any earthquakes having magnitudes in excess of 6.0 are considered dangerous. The largest yet recorded, the Chilean earthquake of 1960, registered 9.4 on the Richter scale. The most powerful recorded in North America, the Alaska quake of 1964, reached 8.4 on the Richter scale.

The Richter Scale





Complete the following:

a) An earthquake of magnitude 8 is _______ times as intense as an earthquake of magnitude 7.

$10^{8-7} = 10^1 = \boxed{10x}$

b) An earthquake of magnitude 7 is _______ times as intense as an earthquake of magnitude 3.

$10^{7-3} = 10^4 = \boxed{10000x}$

c) An earthquake of magnitude 4 is _______ times as intense as an earthquake of magnitude 6.

$10^{4-6} = 10^{-2} = \boxed{\frac{1}{100}x}$

d) An earthquake of magnitude 4.8 is _______ times as intense as an earthquake of magnitude 6.8.

$10^{4.8-6.8} = 10^{-2} = \boxed{\frac{1}{100}x}$

e) The 1960 earthquake in Chile (9.5) was __1000__ times as intense as the 1976 earthquake in Italy (6.5) and __10000__ times as intense as the 1983 earthquake in Colombia (5.5).

$10^{9.5-6.5} = 10^3$

$10^{9.5-5.5} = 10^4$

f) The 1966 earthquake in Turkey (6.9) was __$\frac{1}{100}$__ times as intense as the 1933 earthquake in Japan (8.9).

10

$10^{6.9-8.9} = 10^{-2}$

Comparing Earthquake Intensities

We have seen how to compare the intensity of earthquakes whose magnitude differs by an integer. How would we compare the intensities of the 1960 earthquake in Chile (magnitude 9.5) and the 2005 earthquake in Chile (magnitude 7.8)?

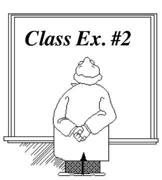
To compare the intensities of magnitude M_1 and M_2, consider the following:

The magnitude of an earthquake is given by the formula $M = \log\left(\frac{I}{I_0}\right)$, where I is the earthquake intensity and I_0 is a reference intensity, e.g., $M_1 = \log\left(\frac{I_1}{I_0}\right)$ and $M_2 = \log\left(\frac{I_2}{I_0}\right)$.

a) Show that $M_1 - M_2 = \log\left(\frac{I_1}{I_2}\right)$.

b) Hence, show that $\frac{I_1}{I_2} = 10^{M_1 - M_2}$.

The intensities of two earthquakes can be compared using $\frac{I_1}{I_2} = 10^{M_1 - M_2}$

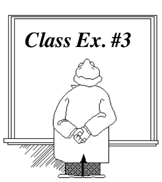


How many times more intense was the 1960 earthquake in Chile (magnitude 9.5) than the 2005 earthquake in Chile (magnitude 7.8)? Answer to the nearest whole number.

$M_1 = 9.5$

$M_2 = 7.8$

$10^{9.5-7.8} = 10^{1.7} \approx \boxed{50\times}$



A major earthquake of magnitude 7.5 is 375 times as intense as a minor earthquake. Find the magnitude, to the nearest tenth, of the minor earthquake.

$375 = 10^{7.5-x}$

$\log 375 = 7.5 - x$

$x = 7.5 - \log 375 \approx \boxed{4.9}$

Loudness of Sound

The loudness of a sound was originally measured in **Bels**, named after Alexander Graham Bell.

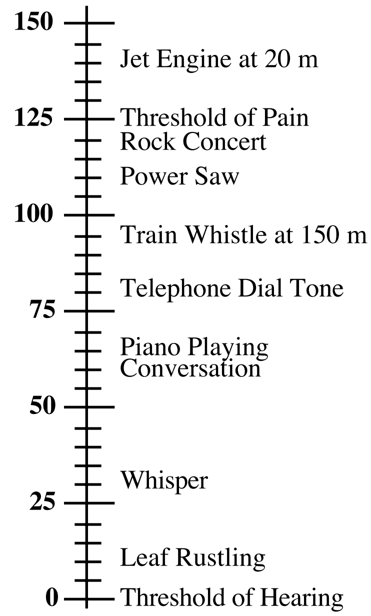
The current unit used is the **decibel (dB)**, which is equal to one tenth of a Bel.

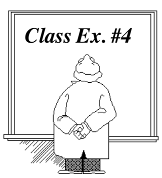
The Bel scale, like the Richter scale, is logarithmic – a difference of 1 Bel, or 10 decibels, corresponds to a factor of ten difference in sound intensity.

A leaf rustling (10 decibels or 1 Bel) is 10 times as loud as the threshold of hearing.

A whisper (30 decibels or 3 Bels) is 10^3, or 1000, times as intense as the threshold of hearing and 10^2, or 100, times as loud as a leaf rustling.

The Decibel (dB) Scale





Complete the following using the chart above.

a) A power saw (120 dB) is 10000 times as loud as a telephone dial tone (80 dB).

$$10^{\frac{120-80}{10}} = 10^4 = 10000$$

b) A jet engine at 20 m (145 dB) is 100 times as loud as the threshold of pain (125 dB).

$$10^{\frac{145-125}{10}} = 10^2$$

c) A whisper (30 dB) is _____ times as loud as a conversation (60 dB).

$$10^{\frac{30-60}{10}} = 10^{-3} = \boxed{\frac{1}{1000}\ x}$$

Comparing Loudness of Sounds

We have seen how to compare the loudness of sounds whose Bels differ by an integer. To compare the loudness of sounds when the Bels are not integers, we suggest the following approach.

The loudness, or intensity, of sound can be measured in dB or Bels. The intensity of two sounds can be compared using the following formulas where I represents sound intensity, and I_0 represents a reference sound intensity (threshold of hearing):

<u>For dB</u>

$$\text{dB} = 10\log\left(\frac{I}{I_0}\right)$$

To compare the loudness, or intensity, of two sounds measured in **decibels**, use the formula

$$\frac{I_1}{I_2} = 10^{\frac{\text{dB}_1 - \text{dB}_2}{10}}$$

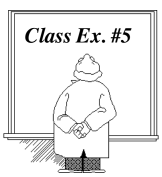
<u>For Bels</u>

$$\text{Bels} = \log\left(\frac{I}{I_0}\right)$$

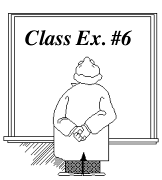
To compare the loudness, or intensity, of two sounds measured in **Bels**, use the formula

$$\frac{I_1}{I_2} = 10^{\text{B}_1 - \text{B}_2}$$

We can derive the above formulas in a way similar to the one used in the previous section.

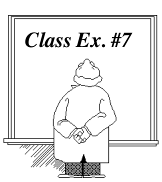


Given that $\text{dB} = 10\log\left(\frac{I}{I_0}\right)$, show that the decibel level for the threshold of pain (125 dB) has a sound intensity $10^{12.5}I_0$.



How many times more intense is the sound of a piano playing (67 dB) than a whisper (22 dB)?

$$10^{\frac{67-22}{10}} = 10^{4.5} \approx \boxed{31622\times}$$

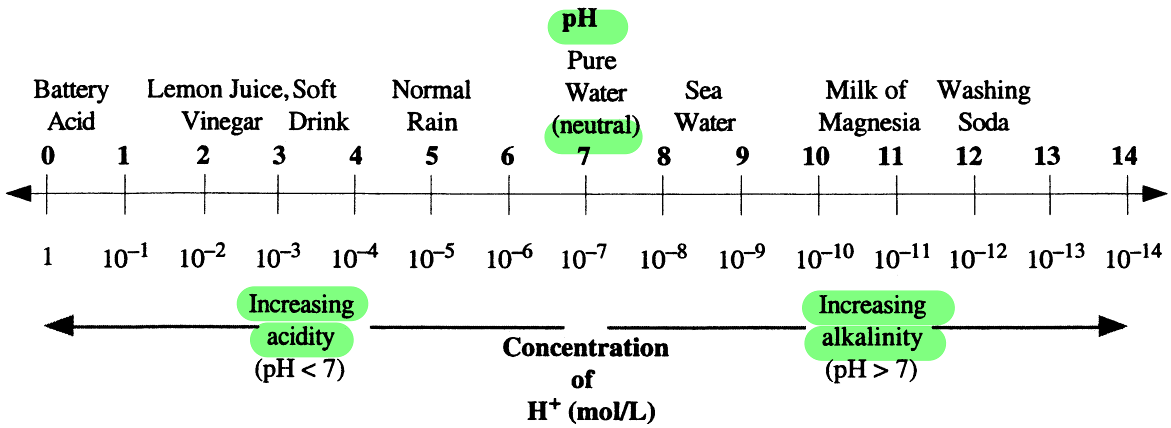


Two telephones in a home ring at the same time with a loudness of 80 decibels each. Does this mean that the total loudness is 160 dB? Explain why or why not using sound intensities and the properties of logarithms.

pH Scale

In 1909, Sören Sörenson, a Dutch chemist, introduced the term **pH**, representing the expression "the power of hydrogen", to measure the extreme wide range of hydrogen ion concentration in substances.

The pH scale measures the range of hydrogen ion concentration by determining the acidity or the alkalinity of a solution. The scale measures from 0 to 14 with values below 7 representing increasing acidity, and values above 7 representing increasing alkalinity. The value 7 represents the neutral level on the pH scale where the solution is neither acidic nor alkaline.

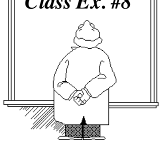


Similar to the Richter scale, the pH scale is logarithmic – a difference in one unit of pH corresponds to a factor of ten difference in intensity. This means that each whole number step represents a ten-fold increase in intensity.

Therefore, if vinegar has a pH of 3, then it is ten times as **acidic** as tomato juice (pH of 4), and one hundred times as acidic as normal rain (pH of 5).

On the other hand, household ammonia (pH of 11.5), is ten times as **alkaline** as milk of magnesia (pH of 10.5) and one thousand times as alkaline as sea water (pH of 8.5).

Class Ex. #8



Complete the following, using the approximate pH values given.

a) Tomato juice is __1000__ times as acidic as pure water. 10^{7-4}

b) Eggs are __10__ times as alkaline as pure water. 10^1

c) Milk of magnesia is __1000__ times as __alkaline__ as blood. 10

d) __Vinegar__ is 100 times as acidic as normal rain. 10

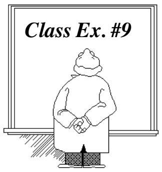
e) Eggs are __$\frac{1}{10000}$__ times as alkaline as washing soda. 10^{8-12}

10^{-4}

Solution	pH
Battery Acid	0.5
Lemon Juice	2.5
Vinegar	3
Tomato Juice	4
Normal Rain	5
Pure Water	7
Blood	7.5
Eggs	8
Milk of Magnesia	10.5
Washing Soda	12

Comparing Acidity and Alkalinity of Solutions

To find how much more **acidic** or **alkaline** one solution is to another, use $10^{pH_1 - pH_2}$.



Class Ex. #9

Pure water, swimming pool water, and sea water have pH levels of 7, 7.5, and 8.4 respectively.

a) How many times as alkaline is sea water than pure water?

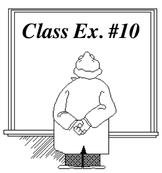
$10^{8.4-7} = 10^{1.4} \approx \boxed{25\times}$

b) How many times as alkaline as swimming pool water is sea water?

$10^{8.4-7.5} = 10^{0.9} \approx \boxed{8\times}$

Formula for pH

The pH of a solution is defined as $pH = -\log [H^+]$, where the H^+ is the hydrogen ion concentration (expressed as moles/litre).



Class Ex. #10

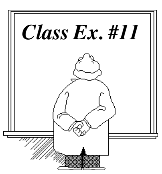
A patient gave a urine sample which was found to have a pH of 5.7. What was the hydrogen ion concentration? Answer in scientific notation using one decimal place.

$pH = -\log [H^+]$

$5.7 = -\log [H^+] \longrightarrow -5.7 = \log [H^+]$

$10^{-5.7} = [H^+]$

$[H^+] = 2.0 \times 10^{-6}$ mol/L



Class Ex. #11

Determine the pH of a solution, to the nearest tenth, if the hydrogen ion concentration is 3.4×10^{-4} mol/L.

$pH = -\log [H^+]$

$= -\log 3.4 \times 10^{-4} = \boxed{3.5}$

Complete Assignment Questions #1 - #11

$\boxed{\#1\text{-}11 \text{ (omit } \#10)}$ + problem sets

Assignment

1. How many times more intense was the 2011 earthquake in Japan (magnitude 9.0) than the 2012 earthquake in Peru (magnitude 6.3)? Answer to the nearest whole number

2. At 2:45 pm on March 11, 2011, a major earthquake of magnitude 9.0 hit the east coast of Japan. Half an hour later, a second earthquake of magnitude 7.9 hit the same region. How many times more intense, to the nearest whole number, was the first earthquake than the second one?

3. An earthquake in Peru had a magnitude of 7.7 on the Richter Scale. The following day a second earthquake with one third of the intensity of the first hit the same region. Determine the magnitude of the second earthquake to the nearest tenth.